# H-2-31-22

Roll No. ..................

## II Semester Examination, 2022

**M.Sc.**

MATHEMATICS

**Paper II**

(Real Analysis-II)

Time : 3 Hours ] [ Max. Marks : 80

**Note :** *All Questions are Compulsory. Question Paper comprises of 3 sections.* ***Section 'A'*** *is Objective type/Multiple Choice questions with no internal Choice.* ***Section 'B'*** *is Short answer type with internal Choice.* ***Section 'C'*** *is Long answer type with internal Choice.*

**Section 'A'** **1 × 10 = 10**

***(Objective Type Questions)***

Choose the correct answer :

**1.** A function $f$ is Riemann intregable on $[a, b]$ iff :

(a) Only $\int_a^{-b} f\,dx$ exist

(b) Only $\int_{-a}^{b} f\,dx$ exist

(c) $\int_{-a}^{b} f\,dx \neq \int_a^{-b} f\,dx$

(d) $\int_{-a}^{b} f\,dx = \int_a^{-b} f\,dx$

P.T.O.



**2.** A function $f \in R(\alpha)$ on $[a, b]$ and $m$, $M$ are lower and upper bounds of the function $f$ respectively, then :

(a) $m\,[\alpha(b) - \alpha(a)] \geq M\,[\alpha(b) - \alpha(a)]$

(b) $m\,[\alpha(b) - \alpha(a)] \leq M\,[\alpha(b) - \alpha(a)]$

(c) $m\,[\alpha(a) - \alpha(b)] \leq M\,[\alpha(a) - \alpha(b)]$

(d) $m\,[\alpha(a) - \alpha(b)] \geq M\,[\alpha(a) - \alpha(b)]$

**3.** The $\sigma$-algebra generated by the family of all open sets in $R$ called :

(a) Measurable sets

(b) Regular Sets

(c) Borel sets

(d) Algebra of sets

**4.** If $A$ and $B$ are two sets with $A \subset B$, then $m^*(A) \leq m^*(B)$. This property is :

(a) Countability

(b) Monotonicity

(c) Countable additivity

(d) Translation invariant

**5.** Consider the following statements :

(i) The Lebsegue measure is complete

(ii) The Lebsegue measure restricted to the $\sigma$-algebra of Borel set is not complete

(a) (i) is true and (ii) is false

(b) (i) is false and (ii) is true

(c) (i) and (ii) both are false

(d) (i) and (ii) both are true

**6.** Cantor set has measures :

(a) $\frac{1}{3}$ (b) 1

(c) 0 (d) $\frac{2}{3}$

**7.** Consider the following statements :

(i) If a function is monotonic, then it has a derivative almost everywhere.

(ii) If a function is bounded variation, then it has derivative almost everywhere.

(a) (i) is true and (ii) is false

(b) (i) is false and (ii) is true

(c) (i) and (ii) both are false

(d) (i) and (ii) both are true





**8.** If $S = \{x : D^{+} f(x) = \infty\}$. Then $S$ has a measure :

(a) 0 (b) 1

(c) $\infty$ (d) None of these

**9.** If $f(x) \in L^2$ and $g(x) \in L^2$, then :

(a) $f(x)\, g(x) \in L^2$ (b) $f(x)\, g(x) \in L^{1/2}$

(c) $f(x)\, g(x) \in L^1$ (d) None of these

**10.** A function of bounded variation need not be :

(a) Bounded

(b) Continuous

(c) Monotonic function

(d) Total variation

**Section 'B'** **4 × 5 = 20**

***(Short Answer Type Questions)***

**Note :** *Attempt one question from each unit.*

**Unit–I**

**1.** Let $f$ be continuous function and $\alpha$ monotonically increasing on $[a, b]$ then prove that $f \in R(\alpha)$ on $[a, b]$.

**Or**

Define rectifiable curve. Let $\gamma : [a, b] \to R^k$ be a curve. If $c \in (a, b)$ then prove that : $\Lambda_\gamma (a, b) = \Lambda_\gamma (a, c) + \Lambda_\gamma (c, b)$.

**Unit–II**

**2.** Prove that the outer measure of an interval is its length.

**Or**

Let $E_n$ be countable collection sets of real numbers. Then show that

$$m^* \bigcup_{n=1}^{\infty} E_n \leq \sum_{n=1}^{\infty} m^*(E_n)$$

**Unit–III**

**3.** State and prove Fatau'slemma.

**Or**

If $A$ and $B$ are disjoint measurable subsets of $E$, then prove that

$$\int_{A \cup B} f = \int_A f + \int_B f$$





**Unit–IV**

**4.** Define four derivatives and find all four Dini Derivative of the following Let $f : R \to R$ be defined by

$$f(x) = \begin{cases} x \dfrac{e^{1/x} - e^{-1/x}}{e^{1/x} + e^{-1/x}}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$$

**Or**

If $f$ is absolutely continuous on $[a, b]$ and $f = 0$ almost everywhere then show that $f$ is a constant function.

**Unit–V**

**5.** Let $1 \leq p \leq \infty$ and let $f, g \in L^p(\mu)$. Then

$$f + g \in L^p(\mu)$$

and $$\| f + g \|_p \leq \| f \|_p + \| g \|_p$$

**Or**

Define $L^P$ space. If $f \in L^P [a, b]$ and $g \leq f$, then $g \in L^P [a, b]$.

## Section 'C' 10 × 5 = 50

***(Long Answer Type Questions)***

### Unit–I

**1.** Explain the "Riemann integral is seen to be a special case of the Riemann-Stieltjes integral." Let $f, \alpha : [a, b] \to R$ be bounded functions and $\alpha$ be monotone increasing. If $P^*$ is *a* refinement of partition $P$ of the interval $[a, b]$, then prove that :

$$L(P, f, \alpha) \leq L(P^*, f, \alpha) \text{ and}$$

$$U(P^*, f, \alpha) \leq U(P, f, \alpha)$$

**Or**

If $f \in R(\alpha_1)$ and $f \in R(\alpha_2)$, then prove that $f \in R(\alpha_1 + \alpha_2)$ and

$$\int_a^b f d(\alpha_1 + \alpha_2) = \int_a^b f\, d\alpha_1 + \int_a^b f\, d\alpha_2$$

### Unit–II

**2.** Define Borel set. Prove that every Borel set in $R$ is measurable.

**Or**

Define measurable function. If $f$ and $g$ be measurable functions on measurable set $E$ then show that $f + g$ is also measurable on $E$.





### Unit–III

**3.** Explain extension of a measure. If $A \in \alpha$ (algebra of sets) then prove that

$$\mu^*(A) = \mu(A)$$

**Or**

State and prove Lebesgue Dominated convergence theorem.

### Unit–IV

**4.** State and prove Vitali's covering Lemma.

**Or**

Let $f$ be a Lebesgue integrable function on $[a, b]$ then the indefinite integral of $f$ is a continuous of bounded variation on $[a, b]$.

### Unit–V

**5.** State and prove Minkowski's inequality.

**Or**

State and prove Riesz-Fischer theorem.

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