H-2-32-22

Roll No. ....

II Semester Examination, 2022

M.Sc.

MATHEMATICS

Paper III

(General and Algebraic Topology)

Time : 3 Hours] [Max. Marks : 80

Note : *All Questions are Compulsory. Question Paper comprises of 3 sections.* ***Section 'A'*** *is Objective type/Multiple Choice questions with no internal Choice.* ***Section 'B'*** *is Short answer type with internal Choice.* ***Section 'C'*** *is Long answer type with internal Choice.*

Section 'A' **1 × 10 = 10**

(Objective Type Questions)

Choose the correct answer :

1. Which of the following space is not first countable ?

(a) Discrete space

(b) (R, U) space

(c) Metric space

(d) Co-countable topology on a uncountable set

P.T.O.

2. Which of the following space is not a Hausdorff space ?

(a) (X, ∞)

(b) (X, I) with only two points

(c) Co-finite topology on an infinite set

(d) (R, U) space

3. Which of the following statement is false ?

(a) The diagonal map is one-one

(b) The product of two connected spaces is C connected space

(c) Projection mappings are open

(d) Projection mappings are closed

4. Which of the following statements is false ?

X topological space (X, y) is compact iff :

(a) every open cover has a finite subcover

(b) every basic open cover has a finite subcover

(c) every sub-basic open cover has a finite subcover

(d) Compactness is a hereditary property

5. Which of the following statement is false ?

(a) Space (R, U) is metrizable

(b) Metrizability is a hereditary property

(c) [0, 1] is not metrizable

(d) Hillbert cube is metrizable

6. Which of the following statement is false ?

If $f_\lambda : X \to Y_\lambda$ for each $\lambda \in \Lambda$, then the evaluation map $e : X \to XY_\lambda$ is :

(a) continuous iff each f_λ is continous

(b) open iff each f_λ is open

(c) one-one iff $\exists = \{f_\lambda : \lambda \in n\}$ distinguishes points

(d) open iff $\exists = \{f_\lambda : \lambda \in \Lambda\}$ distinguishes points and closed sets

7. A net in a discrete toplogical space converges to :

(a) a unique point

(b) at least two points

(c) many points

(d) none of the above

8. Which of the following is not true ?

(a) $\{X\}$ is a filter on X

(b) If F_0 is a non-empty subset of X, Then $\exists = \{F \mid F \supset F_0\}$ is a filter on X

(c) Union of two filters on X is a filter on X

(d) A filter has F, P

9. Which of the following is false ?

(a) The relation of path homotopy is an equivalence relation

(b) The fundamental group of the figure eight is abelian

(c) In a simply connected space X, any two paths having the some initial and final points are path homotopic

(d) The fundamental group of the double torus T_2 is not abelian

10. Which of the following is not true ?

(a) The product of two covering maps is not a covering map in general

(b) The map $p : R \to S'$ given by the equation

$$p(x) = (\cos 2\pi x, \sin 2\pi x)$$

is a covering map

(c) If X is path connected and x_0 and x_1 are two points of X, then $\pi_1(X, x_0)$ is isomorphic to $\pi_1(X, x_1)$

(d) In a simply connected space X, any two paths having the same initial and final points are path homotopic.

Section 'B' **4 × 5 = 20**

(Short Answer Type Questions)

Note : *Attempt one question from each unit.*

Unit–I

1. Prove that the property of being a regular space is a topological property.

Or

Show that the product of two separable spaces is a separable space.

Unit–II

2. State and prove Alexander sub-base lemma.

Or

If $y_0 \in Y$ be a fixed element of Y and let $A = X \times \{y_0\}$. Then prove that π_x/A is an open mapping.

Unit–III

3. State and prove Tychonoff Embedding theorem.

Or

A space X is Tychonoff iff X is homeomorphic to a subspace of a cube.

Unit–IV

4. Prove that every filter on a set X is contained in an ultra filter on X.

Or

Prove that a topological space (X, y) is compact iff each net in X has a cluster point.

Unit–V

5. Define the following :

(i) Covering space

(ii) Fundamental group of a topological space

Or

If $h : (X, x_0) \to (Y, y_0)$ is a homeomorphism of X with Y, then prove that h_* is an isomorphism of $\pi_1(X, x_0)$ with $\pi_1(Y, y_0)$.

Section 'C' **10 × 5 = 50**

(Long Answer Type Questions)

Note : *Attempt one question from each unit.*

Unit–I

1. Prove that $X\{X_\lambda : \lambda \in \Lambda\}$ is Hausdorff iff each co-ordinate space X_λ is Hausdorff.

Or

Prove that $X\{X_\lambda : \lambda \in \Lambda\}$ is regular iff each co-ordinate space X_λ is regular.

Unit–II

2. Prove that $X \times Y$ is connected iff X and Y are connected.

Or

$X\{X_\lambda : \lambda \in \Lambda\}$ is compact iff each X_λ is compact.

Unit–III

3. State & prove Urysohn's Metrization theorem.

Or

State and prove smirnor metrization theorem.

Unit–IV

4. Prove that a topological space (X, Y) is Hausdorff iff every set in X can converge to at most one point.

Or

Prove that a topological space (X, Y) is Hausdorff iff every convergent filter in X has a unique limit.

Unit–V

5. Prove that the fundamental group of the circle is infinite cyclic.

Or

State and prove fundamental theorem of Algebra.

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