H-2-33-22 Roll No. ....

II Semester Examination, 2022

M.Sc.

MATHEMATICS

Paper IV

(Advanced Complex Analysis-II)

Time : 3 Hours] [Max. Marks : 80

Note : *All Questions are Compulsory. Question Paper comprises of 3 sections.* ***Section 'A'*** *is Objective type/Multiple Choice questions with no internal Choice.* ***Section 'B'*** *is Short answer type with internal Choice.* ***Section 'C'*** *is Long answer type with internal Choice.*

Section 'A' 1 × 10 = 10

(Multiple Choice Questions)

Answer the following one sentence :

1. Give an expression of Gauss formula for Γz.
2. Define Riemann zeta function $\zeta(z)$.
3. Define fixed end point homotopic curves.
4. Define complete analytic function.
5. Define mean value property for a continuous function in C.

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6. Define Green's function.
7. Define Rank of an analytic function.
8. Define Laudau's constant.
9. Give statement of Great Picard's theorem.
10. Define Lacunary value of a function $f(z)$.

Section 'B' 4 × 5 = 20

(Short Answer Type Questions)

Note : *Attempt one question from each unit.*

Unit–I

1. Find residue of Γz at poles.

Or

If $|z| \leq 1$ and $p \geq 0$. Then prove that

$$|1 - E_p(z)| \leq |z|^{p+1}$$

Unit–II

2. Find the regions D_1 and D_2 so that the function

$f_2(z) = i\sum_{n=0}^{\infty}\left(\frac{z+i}{i}\right)^n$ is analytic continuation of

$f_1(z) = \int_0^{\infty} e^{-zt}dt$.

Or

Let $\gamma : [0, 1] \to C$ be a path and let $\{(f_t, D_t) : 0 \leq t \leq 1\}$ be an analytic continuation along γ. For $0 \leq t \leq 1$, let $R(t)$ be the radius of convergence of the power series expansion of f_t about $z = \gamma(t)$. Then prove that $R(t) = \infty$ or $R : [0, 1] \to (0, \infty)$ is continuous.

Unit–III

3. Prove that the poisson's Kernel

$$P_r(\theta) = \text{Re}\left(\frac{1 + re^{i\theta}}{1 - re^{i\theta}}\right)$$

Or

If $u : G \to IR$ is a continuous function which has the mean value property, then prove that u is harmonic.

Unit–IV

4. Let $Re\, z_n > -1$, then prove that the series $\Sigma \log(1 + z_n)$ converges absolutely iff the series Σz_n converges absolutely.

Or

If $f(z)$ is an entire function of finite order with $M(r) = \max |f(z)|$ on $|z| = r$ and $n(r)$ be the number of zeros of $f(z)$ in $|z| \leq r$ then prove that

$$\log M(2r) \geq n(r) \log 2$$

Unit–V

5. Suppose g is analytic on $B(0, R)$, $g(0) = 0$, $|g'(0)| = \mu > 0$ and $|g(z)| \leq M$ for all z. Then prove that

$$g[B(0, R)] \supset B\left(0, \frac{R^2\mu^2}{\sigma M}\right)$$

Or

Give statement of Biberbach conjecture.

Section 'C' **10 × 5 = 50**

(Long Answer Type Questions)

Note : *Attempt one question from each unit.*

Unit–I

1. Prove that

$$\sqrt{\pi}\,\sqrt{(2z)} = 2^{2z-1}\sqrt{z}\,\sqrt{\left(z + \frac{1}{2}\right)}.$$

Or

State and prove Runge's theorem.

Unit–II

2. State and prove schwartz's Reflection principle.

Or

Let $\gamma : [0, 1] \to C$ be a path from a to b and let $\{(f_{t_1} D_t) \mid 0 \leq t \leq 1\}$ be an analytic. Continuation along γ. There is a number $\in > 0$ such that if $\sigma : [0, 1] \to C$ is any path from a to b with $| \gamma(t) - \sigma(t) | < \in$ for all t and if $\{(g_t, B_t) : 0 \leq t \leq 1\}$ is any analytic continuation along σ with $[g_0]_a = [f_0]_a$, then prove that $[g_1]_b = [f_1]_b$.

Unit–III

3. Let $D = \{z : | z | < 1\}$ and suppose that $f : \partial D \to R$ is a continuous function. Then prove that there a continuous function $u : \bar{D} \to R$ such that :

(a) $u(z) = f(z)$ for z in ∂D;

(b) u is harmonic in D.

Also u is unique and defined by

$$u\,(re^{i\theta}) = \frac{1}{2\pi}\int_{-\pi}^{\pi} P_r(\theta - t) f(e^{it})\, dt$$

for $0 \leq r < 1$, $0 < \theta \leq 2\pi$.

Or

State and prove Harnack's theorem.

Unit–IV

4. Let (X, d) be a compact metric space and let $\{g_n\}$ be a sequence of continuous functions from X into C such that $\Sigma\, g_n(x)$ converges absolutely and uniformly for x in X. Then prove that the product

$$f(x) = \prod_{n=1}^{\infty} [1 + g_n(x)]$$

Converges absolutely and uniformly for x in X, also there is/an integer n_0 such that

$f(x) = 0$ iff $g_n(x) = -1$ for some

n, $1\ \Sigma\ n \leq n_0$.

Or

State and prove Borel's theorem.

Unit–V

5. State and prove Bloch's theorem.

Or

State and prove little picard's theorem.

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