H-4/23/22

Roll No. ....

IV Semester Examination, 2022

M.Sc.

MATHEMATICS

Paper I

(Functional Analysis-II)

Time : 3 Hours] [Max. Marks : 80

Note : *All questions are compulsory. Question Paper comprises of 3 sections. Section **A** is objective type/multiple choice questions with no internal choice. Section **B** is short answer type with internal choice. Section **C** is long answer type with internal choice.*

SECTION A 1×10=10

(Objective Type/Multiple Type Questions)

Choose the correct answer :

1. Let X and Y be a banach spaces and $T \in B(X, Y)$. It T is onto, then there exists $K > 0$ such that for every $y \in Y$, there exists $x \in X$ such that :

(a) $T_x = y \| x \| \geq K \| y \|$

(b) $T_x = \| y \| x \leq K \| x \|$

(c) $T_x = y \| x \| \leq K \| y \|$

(d) $T_x = y \| x \| \leq K \| x \| \| y \|$

P.T.O.

2. Let N and N' be normed linear space and DCN then a linear transformation $T: D \rightarrow N'$ is closed if and only if G_T is :

(a) Open (b) Closed

(c) Both (a) and (b) (d) None of these

3. A normed linear space X is reflexive, if :

(a) $T(X) = X^{**}$ (b) $T(X) = X$

(c) $T(X) = X^{*}$ (d) $T(X) = X^{***}$

4. Let $\{T_n\}$ be a sequence of compact linear operator from a normed space X into Banach space Y. If $\{T_n\}$ is uniformly operator convergence to T *i.e.*, $\| T_n - T \| \rightarrow 0$ as $n \rightarrow \infty$ then the limit T is ....

(a) Uniform convergence

(b) Compact

(c) Bounded

(d) None of the above

5. Let M be a linear subspace of Hilbert space X then M is closed of and only if :

(a) $M \neq M^{\perp\perp}$ (b) $M = M^{\perp}$

(c) $M \neq M^{\perp}$ (d) $M = M^{\perp\perp}$

6. A normed space is an inner product space if and only if the norm of the normed space satisfy the .... equation.

(a) $\| x + y \| \leq \| x \| + \| y \|$

(b) $|\langle x, y \rangle| \leq \| x \| . \| y \|$

(c) $\| x + y \|^2 + \| x - y \|^2 = 2 \| x \|^2 + 2 \| y \|^2$

(d) None of the above

7. A Banach space X is said to be reflexivive if it is isometrically isomorphic to :

(a) X^* (b) X^{**}

(c) X^{***} (d) All of these

8. If M is a subspace of the Hilbert space H then for any $x \in H$ there exists a unique $y \in M$ and $z \perp M$ such that $x = y + z$ the vector y is called :

(a) Projection of x onto M

(b) Linear of x onto M

(c) Bounded linear of x onto M

(d) None of the above

9. Which is a incorrect statement ?

(a) Every positive operator is self adjoint

(b) Every self adjoint operator is normal

(c) Every normal operator is unitary

(d) Every unitary operator is normal

10. If T is a positive operator on a Hilbert space H then :

(a) $I - T$ is non singular

(b) $I + T$ is non singular

(c) $I + T$ is singular

(d) None of the above

SECTION B **4×5=20**

(Short Answer Type Questions)

Note : Attempt *one* question from each unit.

Unit-I

1. Let X be a Banach space over the field K. If $\{T_n\} \in B(X, Y)$ be a sequence such that $\lim_{n \to \infty} T_n x = Tx$ where $x \in X$ exist then prove that $T \in B(X, Y)$.

Or

Let X and Y are two Banach space and if T is linear transformation from X onto Y then T is continuous if and only if graph of T *i.e.*, T_G is closed.

Unit-II

2. State and prove closed range theorem.

Or

Let X and Y be normed spaces $T : X \rightarrow Y$ is compact linear operator. Suppose that sequence

$\{x_n\}$ in X is weakely convergent salf $x_n \xrightarrow{\omega} x$ then $[T(x_n)]$ is strongly convergent in Y and has the limit $y = T_x$.

Unit-III

3. Let $\{e_1, e_2 ..... e_n\}$ is a finite othonormal set in Hilbert space H if $x \in H$ then $\sum_{i=1}^{n} |x e_i|^2 \leq \|x\|^2$ further $x - \sum (x_1\ e_i)\, e_i \perp e_j$ for each j.

Or

State and prove schwartz inequality.

Unit-IV

4. Let M be a closed linear subspace of a Hilbert space H then $H = M \oplus M^{\perp}$.

Or

Let H is a Hilbert space then show that H^* is also hilbert space with respect to the inner product defined by

$$\langle f_x, f_y \rangle = \langle y, x \rangle$$

Unit-V

5. Prove that $T \in B(X)$ is unitary if and only if it is an isometric isomorphism of X onto itself.

Or

Let A_1 and A_2 are self adjoint operators α and β are real numbers then show that $\alpha A_1 + \beta A_2$ also self adjoint.

SECTION C **10×5=50**

(Long Answer Type Questions)

Note : Attempt *one* question from each unit.

Unit-I

1. State and prove uniform boundedness theorem.

Or

Let X and Y are Banach space and T is linear transformation from X onto Y then T is an open mapping.

Unit-II

2. Let N be a arbitrary normed linear space to each vector $x \in N$ then $\exists$ scalar value function F_x define on N^{**} such that $F_x(f) = f(x)\ \forall f \in N^*$ then show that F_x is continuouse linear function and the mapping $\psi : x \to F_x$ is an isometric isomorphism.

Or

State and prove Hahn Banach theorem for normed linear space.

Unit-III

3. If M and N be closed linear subspace of hilbert space H such that $M \perp N$ and $M \cap N = \{0\}$, then so that $M + N$ is also closed linear subspace of H.

Or

A closed convex subset C of a Hilbert space H contains a unit vector of smallest Norm.

Unit-IV

4. State and prove Riesz representation theorem.

Or

Let $T : X \to Y$ be a linear operator. If T is compact so its adjoint operator $T^X : Y' \to X'$.

Unit-V

5. Let S and T be elements of class of normal operator N (H) and suppose that $ST^* = T^*S$ then $S + T$ and ST are in $N(H)$.

Or

Let H be a complete Hilbert space and $T \in B(H)$ then the following statements are equivalent :

(a) T is normal

(b) T^* is normal

(c) $\| T^\forall x \| = \| T_x \| \ \forall\ x \in H$

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