H-4/24/22

Roll No. .....

IV Semester Examination, 2022

M.Sc.

MATHEMATICS

Paper II

(Partial Differential Equations Mechanics-II)

Time : 3 Hours] [Max. Marks : 80

Note : *All questions are compulsory. Question Paper comprises of 3 sections. Section **A** is objective type/multiple choice questions with no internal choice. Section **B** is short answer type with internal choice. Section **C** is long answer type with internal choice.*

SECTION A **1×10=10**

(Objective Type/Multiple Type Questions)

Choose the correct answer :

1. For any multi-index $\alpha = (\alpha_1, \alpha_2 \ldots \alpha_n)$ and any $x = (x_1, x_2 \ldots x_n) \in R^n$

(a) $x^\alpha = x_1^{\alpha 1} x_2^{\alpha 2} \ldots x_n^{\alpha n}$

(b) $x^\alpha = x_1^{\alpha 1} + x_2^{\alpha 2} + \ldots + x_n^{\alpha n}$

(c) Both (a) and (b)

(d) None of (a) and (b)

P.T.O.

2. The role of independent variable and components of gradient of solution are interchanged in :

(a) Cole Hopf Transform

(b) Fourier Transform

(c) Legendre Transform

(d) Hodograph Transform

3. A dynamical system is conservative if :

(a) it is scleronomic

(b) forces are derivable from potential

(c) both (a) and (b)

(d) none of the above

4. The numbers of generalized coordinates in a simple pendulum with a rigid support are :

(a) 1 (b) 2

(c) 0 (d) None of these

5. In Euler Poisson equation, the function depends on :

(a) more than one dependent function

(b) higher order derivatives

(c) more than one independent variable

(d) none of the above

6. The value of $[H, H]$ is :

(a) 1 (b) q

(c) p (d) 0

7. For a contact transformation :

(a) $\frac{\partial q_i}{\partial Q_j} = -\frac{\partial P_j}{\partial p_i}$ (b) $\frac{\partial q_i}{\partial P_j} = \frac{\partial Q}{\partial p_i}$

(c) $\frac{\partial p_i}{\partial Q_j} = \frac{\partial P_j}{\partial q_i}$ (d) $\frac{\partial p_i}{\partial P_j} = \frac{\partial Q_i}{\partial q_i}$

8. The function P (analogous to Lagrangian) in Jacobi's equation is given by :

(a) $2\sqrt{G(h - V)}$ (b) $2\sqrt{G(h + V)}$

(c) $2\sqrt{T(h - V)}$ (d) $2\sqrt{T(h + V)}$

9. Value of $\{cu, v\}_{q, p}$ is :

(a) $c\{u, V\}_{q, p}$ (b) $\frac{1}{c}\{u, v\}_{q, p}$

(c) $\{u, cv]_{q, p}$ (d) $\{u, v\}_{q, p}$

10. The equation below is known as

$$H\left(q_i, \frac{\partial f_2}{\partial q_i}, t\right) + \frac{\partial f_2}{\partial t} = 0$$

(a) Hamilton's principle

(b) Hamilton Canonical equation

(c) Hamilton Jacobi equation

(d) Hamilton Characteristic equation

SECTION B **4×5=20**

(Short Answer Type Questions)

Unit-I

1. If $f = \sum_{\alpha} f_\alpha x^\alpha$ converges for $|x| < r$ and $0 < s\sqrt{n} < r$, then prove that f has a majorant for $|x| < \frac{s}{\sqrt{n}}$.

Or

Explain geometric optics.

Unit-II

2. Explain the classification of constraints.

Or

Explain generalized coordinates and generalized force.

Unit-III

3. Define Poisson Bracket. If $L = \frac{1}{2}q^{\cdot 2} - qq^{\cdot} + q^2$, then find the value of $[p, q^{\cdot 2}]$.

Or

Find the extremals of the isoprimetric problem $I[y(x)] = \int_{x_0}^{x_1} y'^2\, dx$ given that $\int_{x_0}^{x_1} y\, dx = c$, a constant.

Unit-IV

4. Prove that the transformation $q = \sqrt{\frac{2P}{k}} \sin Q$, $p = \sqrt{2Pk} \cos Q$ is canonical, k being a constant. Also find the generating function.

Or

Explain Poincare-Cartan integral invariant.

Unit-V

5. Prove that properties of lagrange bracket :

(i) $\{q_i, q_j\} = 0$ (ii) $\{q_i, p_j\} = \delta_{ij}$

Or

Define Hamilton's Principal function and Hamilton's characteristic function.

SECTION C **10×5=50**

(Long Answer Type Questions)

Unit-I

1. Find solution of Burger's equation using Cole-Hopf transformation.

Or

Explain similarity solutions using plane and travelling waves through example.

Unit-II

2. Derive Hamilton's canonical equations using lagrange's equation.

Or

A particle is constraint to move on the surface of cylinder of radius 'a' under the action of gravity. Find the lagrange's equation of motion.

Unit-III

3. Explain and solve the problem of minimal surface of revolution.

Or

Prove the Poisson identity :

$$[u\,(v, w)] + [v\,(w, u)] + [w\,(u, v)] = 0$$

Unit-IV

4. Derive the lagrange's equation for holonomic, conservative system from hamilton's principle.

Or

Derive the principle of least action.

Unit-V

5. Prove the invariance of lagrange brackets under Canonical transformation.

Or

Explain the Hamilton-Jacobi equation for Hamilton's principal function.

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