

Annual Examination, 2022**B.C.A. Part II**

B.C.A.-201

Paper I

(Calculus and Differential Equations)

Time : 3 Hours]

[Max. Marks : 80

Note : *Attempt any two parts from each unit. All questions carry equal marks.*

Unit-I

1. (a) Evaluate $\lim_{x \rightarrow 0} \left(\frac{e^{1/x}}{e^{1/x} + 1} \right)$, if it exists.

(b) Test for continuity of the function at $x = 1$

$$f(x) = \begin{cases} x^2 - 1, & x \geq 1 \\ 1 - x, & x < 1 \end{cases}$$

(c) If $f(x) = \begin{cases} x \sin \frac{1}{x}, & \text{when } x \neq 0 \\ 0, & \text{when } x = 0 \end{cases}$

Show that the function f is continuous at $x = 0$ but not differential at $x = 0$.

Unit-II

2. (a) Differentiate $\frac{\tan x + \cot x}{\log x}$ w.r.t. x .

(b) Find $\frac{dy}{dx}$ when $x = a \cos^3 t$, $y = a \sin^3 t$.

(c) If $x = f(t)$, $y = g(t)$ then prove that

$$\frac{d^2y}{dx^2} = \frac{x'y'' - y'x''}{(x')^3} \text{ where } x' = \frac{dx}{dt}, y' = \frac{dy}{dt}.$$

Unit-III

3. (a) Evaluate $\int \frac{x^3 \tan^{-1} x^4}{1+x^8} dx$.

(b) Evaluate $\int \frac{1}{(4x^2 + 4x + 5)} dx$.

(c) Prove that $\int x^3 e^{x^2} dx = \frac{1}{2} e^{x^2} (x^2 - 1)$.

Unit-IV

4. (a) Evaluate $\int_0^\pi \sin^3 x dx$.

(b) Evaluate $\int_2^4 \frac{2x}{x^2 + 1} dx$.

(c) If $f(x)$ is of the form $f(x) = a + bx + cx^2$ then show that

$$\int_0^1 f(x) dx = \frac{1}{6} \left\{ f(0) + 4f\left(\frac{1}{2}\right) + f(1) \right\}$$

Unit-V

5. (a) Show that $y = cx + \frac{a}{c}$ is a solution of the

$$\text{differential equation } y = x \frac{dy}{dx} + \frac{a}{\left(\frac{dy}{dx}\right)}.$$

(b) Solve $\frac{dy}{dx} = -y + 1$

(c) Solve the following differential eqn.

$$\frac{dy}{dx} = e^{x-y} + x^2 e^{-y}$$

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