G-1/180/22

Roll No. ....

I Semester Examination, January 2022

M.Sc.

MATHEMATICS

Paper V

(Advanced Discrete Mathematics-I)

Time : 3 Hours] [Max. Marks : 80

Note : *All questions are compulsory. Question Paper comprises of 3 Sections. Section A is objective type/multiple choice questions with no internal choice. Section B is short answer type with internal choice. Section C is long answer type with internal choice.*

SECTION A **1×10=10**

(Objective Type/Multiple Choice Questions)

Choose the correct answer :

1. If $\sim (p \rightarrow q)$ choose the correct answer :

(a) $-p \wedge q$ (b) $-p \vee q$

(c) $p \wedge -q$ (d) $p \vee q$

P.T.O.

2. Negation of the statement $\exists x \, P(x) \vee \forall y \, Q(y)$ is :

(a) $\forall x \, P(x) \wedge \forall y \, Q(x)$

(b) $\forall x \, P(x) \vee \exists y \, Q(x)$

(c) $\forall x \sim P(x) \vee \exists y \sim Q(x)$

(d) $\forall x \sim P(x) \wedge \exists y \sim Q(x)$

3. Let f: S $\rightarrow$ T and (S, *) and (T, Δ) are semigroups then f is isomorphism if :

(a) f is one-one

(b) f is onto

(c) f is homomorphism

(d) All of the above

4. Let g be a semigroup homomorphism from semigroup (S, *) to semigroup (T, $\oplus$) if $a \in$ S is an idempotent element then $g(a)$:

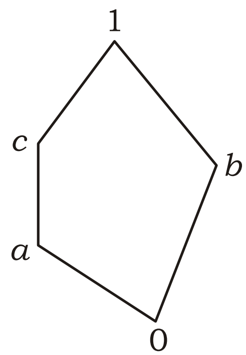
(a) g(a) is not idempotent element

(b) g(a) is an ordinary element

(c) g(a) is also idempotent element

(d) None of the above

5. The lattice of the form :

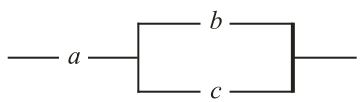


the complement b is :

(a) Only a (b) Only c

(c) Both (a) and (b) (d) None of these

6. The switching circuits :

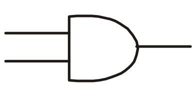
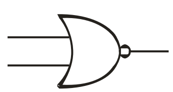


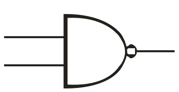
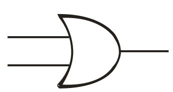
equivalent to :

(a) $a.(b + c)$ (b) $a.b + c$

(c) $a + b.c$ (d) $a.b + b.c$

7. Which one is NAND gate ?

(a)  (b) 

(c)  (d) 

8. In a Boolean algebra (B + ∴ ′)

$$(a + b').(a' + b).(a' + b') =$$

(a) $a.b'$ (b) $a'b'$

(c) $a'.b$ (d) $a.b$

9. The value of complete conjunctive normal form of a Boolean function is :

(a) 1 (b) 0

(c) Both (a) and (b) (d) None of these

10. A and B are non-terminals and a and b are terminals the production $A \to a$, $B \to aA$ is of the type :

(a) regular grammar

(b) context free grammar

(c) context sensitive grammar

(d) none of the above

SECTION B 5×4=20

(Short Answer Type Questions)

Note : *Answer the following questions.*

Unit-I

1. Prove that $\sim(p \wedge q) \to \{\sim p \vee (\sim p \vee q)\} \equiv \sim p \vee q$ without constructing truthtable.

Or

If g is a homomorphism from a commutative semigroup (S, *) onto a semigroup (T, $\oplus$) then show that (T, $\oplus$) is also commutative semigroup.

Unit-II

2. For any commutative monoid (M, *) the set of idempotent element of M forms a submonoid.

Or

Explain direct product of semigroups.

Unit-III

3. L = {1, 2, 3, 4, 6, 12}, (L, '1'), $a \vee b$ = LCM(a, b), $a \wedge b$ = hcf (a, b) show that (L, '1') is a lattice but it is not complemented lattice. ('1' stands for divides).

Or

Show that for Boolean algebra (B + ,· ;) :

$$(a+b).(b+c).(c+a) = a.b + b.c + c.a$$

where $a, b, c \in$ B.

Unit-IV

4. Draw the switching circuit of the function :

$$F(x, y, z) = x.y'.(z+x) + y.(y'+z)$$

and replace it by simplex one.

Or

Draw the logic circuit for the following expression :

(i) $x.y + z.y'$

(ii) $(x+y).(x'+y'+z).(y'.z)$.

Unit-V

5. Explain the regular and contex free grammar.

Or

Construct a grammar for the language :

$$L = \left\{a^i b^{2i};\ i \geq 1\right\}$$

SECTION C **10×5=50**

(Long Answer Type Questions)

Note : *Answer the following questions.*

Unit-I

1. (a) Show that :

(7P ∧ (7Q ∧ R)) ∨ (Q ∧ R) ∨ (P ∧ R) ⇔ R

(b) Show that :

7(P ∧ Q) → (7P ∨ (7P ∨ Q) ⇔ (7P ∨ Q)

Or

Show that the set N of natural number is a semigroup under the operation x*y = max {x, y}. Is it a monoid ?

Unit-II

2. Let (S, *) and (T, 0) be semigroup. If $f : S \rightarrow T$ is a semigroup homomorphism then show that (T, 0) is isomorphic to some quotient semigroup of (S, *).

Or

Define the congruence relation on the semigroup and show that the equivalence relation R defined by a R b if and only if $a \equiv b \pmod 2$ on the semigroup (I, +) of integers under addition is a congruence relation on (I, +).

Unit-III

3. Show that every chain is a distribution lattice.

Or

Show that in a distributive lattice the complement of each element is unique and also show that by an example if the complement of any element of a lattice is not unique then it is not distributive lattice.

Unit-IV

4. Write the following functions in to conjunctive normal form :

$$f(x, y, z) = (x + y + z).(xy + x'z)'$$

Or

Use the Karnaugh map representation find minimal sum of products expression of the function :

$$f(a,b,c,d) = \sum(0,5,7,8,12,14)$$

Unit-V

5. Let A = {0, 1}. Show that the following expressions are regular expression over A.

(i) 0*(0 + 1)*

(ii) 00*(1 + 0)*1

(iii) (01)*(01 + 1*)

Also find regular sets corresponding to these regular expression.

Or

Construct the grammar for the language $L = \{a^n \, b \, a^m \mid m, n \geq 1\}$ and for the string $a^4 \, b \, a^5$ write the derivation.

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