G-1/176/22 Roll No. ....

I Semester Examination, January 2022

M.Sc.

MATHEMATICS

Paper I

(Advanced Abstract Aglebra-I)

Time : 3 Hours] [Max. Marks : 80

Note : *All questions are compulsory. Question Paper comprises of 3 Sections. Section A is objective type/multiple choice questions with no internal choice. Section B is short answer type with internal choice. Section C is long answer type with internal choice.*

SECTION A 1 × 10 = 10

1. Define equivalence of two composition series.
2. Write composition series for $\frac{Z}{(18)}$.
3. Find the degree of $Q(\sqrt{2}, \sqrt{3})$ over Q.
4. Define transcendental extensions.
5. Give an example of finite field.

P.T.O.

6. Determine a minimal polynomial over Q of $\sqrt{(-1)+\sqrt{2}}$.
7. [C : R] = ?
8. State fundamental theorem of algebra.
9. Define radical extension.
10. Is $x^7 - 10x^5 + 15x + 5$ solvable by radicals.

SECTION B 4 × 5 = 20

(Short Answer Type Questions)

Note : *Answer the following questions.*

Unit-I

1. A group has order 7^5, is it nilpotent.

Or

Show that the group of all upper triangular matrices of the form $\begin{bmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{bmatrix}, a, b, c \in R$ is solvable.

Unit-II

2. Show that any polynomial of degree 2 or 3 over F[x] is reducible if and only if it has a root in F.

Or

Is $Q(5\sqrt{7})$ normal over Q ?

Unit-III

3. Show that the multiplicative group of non-zero elements of a finite field is cyclic.

Or

If F is a algebraically closed then show that every polynomial in F[x] of positive degree factors completely in F[x] into linear factors.

Unit-IV

4. Show that $G\left(\frac{Q(\alpha)}{Q}\right), \alpha^5 = 1$ is isomorphic to cyclic of order 4.

Or

Show that $K = E_{G(E/K)}$ where $F \subseteq K \subseteq E$ are fields and E Galois extension of F.

Unit-V

5. Write Eisenstein criteria for solvability of a polynomial over Q.

Or

Write Descarte's rule of signs to find the nature of roots of any polynomial.

SECTION C **10×5=50**

(Long Answer Type Questions)

Note : *Answer the following questions.*

Unit-I

1. State fundamental theorem of arithmetic and prove it with the help of Jordan Holder theorem.

Or

Define nilpotent group and show that if $H_1, H_2, \ldots\ldots\ldots H_n$ are nilpotent groups then $H_1 \times H_2 \times \ldots\ldots\ldots \times H_n$ is also nilpotent.

Unit-II

2. State and prove Gauss's Lemma.

Or

Define simple extension and show that finite separable extension is simple extension.

Unit-III

3. Define perfect field and prove that in a finite field any element can be written as the sum of two squares.

Or

Show that the prime field of a field is either isomorphic to Q or to $\frac{Z}{(P)}$, p-prime.

Unit-IV

4. If $F \subseteq K \subseteq E$ are fields such that E is Galois extension of F then show that K is normal extension of F iff $G\left(\frac{E}{K}\right)$ is a normal subgroup of $G\left(\frac{E}{F}\right)$.

Or

Show that the Galois group of $x^4 + 1 \in Q[x]$ is the Klein four groups.

Unit-V

5. Prove that :

If $f(x) \in F[x]$ is solvable by radicals over F if and only if its splitting field E over F has solvable Galois group $G\left(\frac{E}{F}\right)$.

Or

Show that the Galois group of a monic irreducible polynomial. Over Q of degree $p^{\text{(prime)}}$, $f(x) \in F[x]$ has exactly two normal roots in C, is isomorphic to S_p.

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