G-1/185/22 Roll No. ....

I Semester Examination, January 2022

M.Sc.

PHYSICS

Paper I

(Mathematical Physics)

Time : 3 Hours] [Max. Marks : 80

Note : *All questions are compulsory. Question Paper comprises of 3 Sections. Section A is objective type/multiple choice questions with no internal choice. Section B is short answer type with internal choice. Section C is long answer type with internal choice.*

SECTION A **1×8=8**

(Objective Type/Multiple Choice Questions)

Note : *Answer the following questions.*

1. In unitary matrix A^+ is..................

2. Two vectors X_1 and X_2 are called orthogonal vectors if................

P.T.O.

3. $J_{\frac{-1}{2}}(x) =$ ....

4. $\int_{-1}^{+1} P_n^2(x)dx =$ ....

5. $w = \log z$ is analytic everywhere except at $z =$ ....

6. The CR-equations for $f(z) = u(x, y) + iv(x, y)$ to be analytic are ....

7. Inverse Laplace's transform of $(p+2)^{-2}$ is ....

8. $F^*(-w)$ ....

SECTION B **6×4=24**

(Short Answer Type Questions)

Note : *Answer the following questions.*

Unit-I

1. Write any *five* properties of an orthogonal matrix.

Or

Write properties of eigen values.

Unit-II

2. To prove Laguerre Rodrigues Formula :

$$L_n(x) = \frac{e^x}{n!}\frac{d^n}{dx^n}(x^n e^{-x})$$

Or

To prove :

$$e^{2tx-t^2} = \sum_{n=0}^{\infty}\frac{t^n}{n!}H_n(x)$$

Unit-III

3. Describe about Cauchy's Integral Theorem.

Or

Describe about Cauchy's Integral Formula.

Unit-IV

4. Find the Laplace's transform of $\frac{1-e^t}{t}$.

Or

Write the Linear property of Fourier's transform.

SECTION C **12×4=48**

(Long Answer Type Questions)

Note : *Answer the following questions.*

Unit-I

1. (a) Find the characteristic equation of the matrix :

$$A = \begin{bmatrix} 1 & 2 & -2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$$

(b) Show that the matrix A, defined as under, is orthogonal :

$$[A] = \frac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ -2 & 2 & -1 \end{bmatrix}$$

Or

(a) Show that the matrix [A] as given below is Unitary :

$$A = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} \\ -\frac{i}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$$

(b) Find the eigen values of the matrix :

$$A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$$

Unit-II

2. Solve the Legendre's differential equation :

$$\left(1-x^2\right)\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+n(n+1)y=0$$

(n = constant).

Or

Solve the Laguerre's differential equation :

$$x\frac{d^2y}{dx^2}+(1-x)\frac{dy}{dx}+\lambda y=0$$

(λ = constant).

Unit-III

3. (a) Show that the function $f(z)=\sqrt{(|xy|)}$ is not regular at the origin although the Cauchy-Riemann equation are satisfied at that point.

(b) If $f(z)$ is an analytic function of $|z|$. Prove that :

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)|f(z)|^2=4\,|f'(z)|^2$$

Or

Evaluate the following contour Integration :

(a) $\int_0^{\infty}\frac{1-\cos x}{x^2}dx$

(b) $\int_{-\infty}^{+\infty}\frac{x^2dx}{\left(x^2+a^2\right)^3}$.

Unit-IV

4. (a) Find the Laplace's Transform $\frac{\sin 2t}{t}$.

(b) Find the inverse transform of $\frac{p}{\left(p^4+4a^4\right)}$.

Or

(a) Find the Fourier transform of the function :

$$f(x)=\begin{cases}1-x^2, & \text{for} \quad |x|\le 1\\ 0\ , & \text{for} \quad |x|\ge 1\end{cases}$$

(b) Find Fourier series to represent $f(x)=x^2-2$ in interval $-2<x<2$.

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