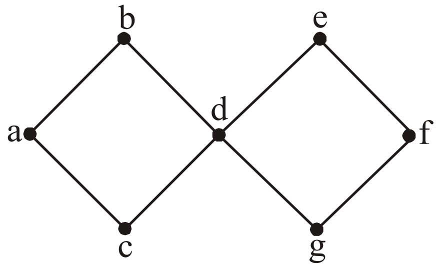
(b) Show that a tree with n vertices has $(n-1)$ edges.

Or

(a) Use BFS algorithm to find a spanning tree of graph G.



(b) Show that in any binary tree T on n vertices, the number of pendent vertices is equal to $(n+1)/2$.

✪ ✪ ✪ ✪ ✪ c ✪ ✪ ✪ ✪ ✪

G-1/127/21 Roll No................

M.Sc. I Semester Examination, April-2021

INFORMATION TECHNOLOGY

Paper III

(Mathematical Foundations of Computer Science)

Time : 3 Hours] [Maximum Marks : 100

Note : *All questions are compulsory. Question Paper comprises of 3 sections.* **Section A** *is objective type/Multiple Choice questions with no internal choice.* **Section B** *is short answer type with internal choice.* **Section C** *is long answer type with internal choice.*

SECTION 'A' **1×10=10**

(Multiple Type Questions)

Choose the correct answer :

1. What is the cartesian product of A = {1, 2} and B = {a, b} ?

(a) {(1, a), (1, b), (2, a), (2, b)}

(b) {(1, 1), (2, 2) (a, a), (b, b)}

(c) {(1, a), (2, a), (1, b), (2, b)}

(d) (1, 1), (a, a), (2, a), (1, b)}

defined by xRy if $x^y = y^x$, where $x, y \in$ I, then is the relation R an equivalence relation ?

Or

(a) If A = {1, 2}, B = {2, 3} and C = {3, 5}, then find $(A \times B) \cap (A \times C)$.

(b) Obtain the principal disjunctive normal form of the following :

(i) $p \Rightarrow q$

(ii) $q \vee (p \vee \sim q)$.

2. (a) Prove that the complement of each element of Boolean algebra B is unique.

(b) Design the circuit for the following polynomials $x + y\,(z + st) + uv$.

Or

(a) Replace the switching function

$$F(x, y, z) = x.y.z + x.y'.z + x'.y'.z$$

by a simpler switching circuit.

(b) Prove that a lattice (L, ≤) is modular if and only if $(a \wedge b) \vee (a \wedge c) = a \wedge (b \vee (a \wedge c))$ for all $a, b, c \in$ L.

3. (a) Define language. Prove the following identities :

$(1 + 00^*1) + (1 + 00^*1)(0 + 10^*1)^*(0 + 10^*1) = 0^*1(0 + 10^*1)^*$

8. The........................ of a graph G consists of all vertices and edges of G.

(a) line graph (b) eulerian circuit

(c) edge graph (d) path complement graph

9. An undirected graph G which is connected and acyclic is called................

(a) Forest (b) Bipartite graph

(c) Tree (d) Cyclic graph

10. In preorder traversal of a binary tree the second step is.......................... .

(a) traverse right subtree and visit the root

(b) visit the root

(c) traverse the right subtree

(d) traverse the left subtree

SECTION 'B' **6×5=30**

(Short Answer Type Questions)

Note : *Answer the following questions.*

1. List all the subsets of the set A = {a, b, c}.

Or

Prove that $(p \Leftrightarrow q) \wedge (q \Leftrightarrow r) \Rightarrow (p \Leftrightarrow r)$ is a tautology.

2. Prove that in a distributive lattice, if an element has a complement then this complement is unique.

Or

For any two elements a and b of a Boolean algebra B, then,

(i) $a + (a.b) = a$

(ii) $a.(a + b) = a$

3. Show that if every element of a group (G,0) be its own inverse, then it is an abelian group.

Or

Let $L_1 = \{x, xy, x^2\}$ and $L_2 = \{y^2, xyz\}$ be language over $A = \{x, y\}$. Find the following :

(i) L_1L_2

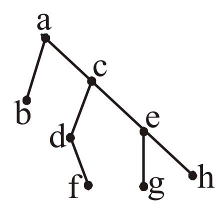
(ii) L_2^2

4. Define Graph. Explain the types of graph with example.

Or

Show that the maximum number of edges in a simple graph with n vertices is $n(n-1)/2$.

5. Consider the rooted tree



(a) What is the root of T ?

(b) Find the leaves and the internal vertices of T.

(c) What are the levels of c and e.

(d) Find the children of c and e.

(e) Find the decendants of the vertices a and c.

Or

Discuss the applications of trees in computer science.

SECTION 'C' **12×5=60**

(Long Answer Type Questions)

Note : *Answer the following questions in 500 words.*

1. (a) If A, B, C are any three non-empty sets, then prove that

$$(A - B) \times C = (A \times C) - (B \times C).$$

(b) If I is the set of non-zero integers and a relation R is

2. $p \wedge q \Rightarrow p \vee q$ is a ....

(a) Tautology (b) Contradiction

(c) Contigency (d) None of the above

3. Evalute the expression :

$$(X + Z)(X - XZ') + XY + Y$$

(a) $XY + Z'$ (b) $Y + XZ' + Y'Z$

(c) $X'Z + Y$ (d) $X + Y$

4. If every two elements of a poset are comparable then the poset is called :

(a) Subordered poset (b) Totally ordered poset

(c) Sub lattice (d) Semigroup

5. A group (M, $*$) is said to be abelian if.................

(a) $(y * x) = (x + y)$ (b) $(x + y) = (y + x)$

(c) $(x * y) = (y * x)$ (d) $(x + y) = x$

6. The polynomial $f(x) = x^3 + x + 1$ is a reducible.

(a) True (b) False

7. In a...................... the vertex set and the edge set are finite sets.

(a) finite graph (b) connected graph

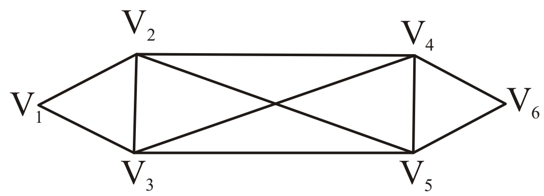
(c) infinite graph (d) bipartite graph

(b) If R is a ring such that $a^2 = a \ \forall \ a \in R$ prove that $a + a = 0 \ \forall \ a \in R$, *i.e.*, each element of R is its own additive inverse.

Or

Define field. Show that every field is an integral domain.

4. (a) Find the degree of each vertex of the following graph :



(b) Prove that the sum of the degree of all vertices in a graph G is equal to twice the number of edges in G.

Or

Show that every self-complementary graph has $4k$ or $4k+1$ vertices.

5. (a) Find the minimal spanning tree of the given weighted graph.