Or

Determine the maximum and minimum value of the function

$$f(x, y) = x^2 + y^2 + \frac{2\sqrt{3}}{3}\, xy$$

subject to the constraints $4x^2 + y^2 = 1$.

5. Suppose W is k-form in an open set $E \subset R^n$, ϕ is a k-surface in E_1 with parameter domain $D \subset R^k$ and Δ is the k-surface in R^k, with parameter domain D, defined by $\Delta(u) = u(u \in D)$, then

$$\int_{\phi} W = \int_{\Delta} W\phi$$

Or

Write a short note on differential forms.

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M.Sc. I Semester Examination, April-2021

MATHEMATICS

Paper II

(Real Analysis-I)

Time : 3 Hours] [Maximum Marks : 80

Note : *All questions are compulsory. Question Paper comprises of 3 sections.* ***Section A*** *is objective type/Multiple Choice questions with no internal choice.* ***Section B*** *is short answer type with internal choice.* ***Section C*** *is long answer type with internal choice.*

SECTION 'A'

(Objective Type Questions)

Choose the correct answer : **1 × 10 = 10**

1. The sequence $\{f_n\}$ where $f_n(x) = nx(1-x)^n$ on [0, 1] does :

(a) converge (b) not converge

(c) uniformly converge (d) not uniformly converge

2. Teh limit function of uniformly convergent sequence of continuous function is itself :

(a) continuous (b) uniformly convergent

(c) convergent (d) not continuous

3. The power series $\sum_{n=1}^{\infty} nx^{n-1}$ has radius of convergence equal to :

(a) $\frac{1}{2}$ (b) 0

(c) 1 (d) -1

P.T.O.

4. Every Cauchy sequence has a :

(a) convergent subsequence

(b) increasing subsequence

(c) decreasing subsequence

(d) positive subsequence

5. Compute the Jacobian $\frac{\partial(x, y)}{\partial(u, v)}$ of the coordinate transformation $x = u^2 - v^4,\ y = uv$:

(a) $2u^2 + 4v^4$ (b) $xu - yv$

(c) $2u^2$ (d) $3u^2 + 7v^6$

6. If $f(x,y) = x^2 + y^2 - xy + 2$, then :

(a) $f(x, y) = -f(y, x)$ (b) $f(x, y) = f(y, x)$

(c) $f(x, y) = -f(x, y)$ (d) $f(x, y) = f(y, x^2)$

7. Sum of the series $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \ldots\ldots\ldots$ is :

(a) $\log 2$ (b) $\frac{3}{2}\log 2$

(c) $\frac{1}{2}\log 2$ (d) $\log 3$

8. Every continuous function on $[a, b]$ is Riemann integrable if :

(a) $U(P, f) - L(P, f) > \varepsilon$ (b) $U(P, f) - L(P, f) \leq \varepsilon$

(c) $U(P, f) - L(P, f) < \varepsilon$ (d) $U(P, f) - L(P, f) \geq \varepsilon$

SECTION 'C' 10 × 5 = 50

(Long Answer Type Questions)

Note : *Answer the following questions.*

1. State and prove Dirichlet's test for uniform convergence.

Or

Suppose $C_n \geq 0$ for $n = 1, 2, 3 \ldots\ldots$, ΣC_n converges, $\{S_n\}$ is a sequence of distinct point in (a, b) and

$$\alpha(a) = \sum_{n=1}^{\infty} C_n I\,(x - S_n).$$

Let f be continuous on $[a, b]$. Then prove that :

$$\int_a^b f d_\alpha = \sum_{n=1}^{\infty} C_n f(S_n)$$

2. State and prove Tauber's theorem.

Or

State and prove Riemann's theorem on rearrangement of series.

3. Let E be an open set in R^n, f maps into R^m, f be differentiable at $x_0 \in E$, g map an open set containing $f(E)$ into R^k and g be differentiable at $f(x_0)$. Then the mapping F of E into R^k defined by $F(x) = g(f(x))$ is differentiable at x_0 and $F'(x_0) = g'(f(x_0))f'(x_0)$.

Or

State and prove the Chain rule.

4. Prove that $\frac{\partial(u, v, w)}{\partial(x, y, z)} \times \frac{\partial(x, y, z)}{\partial(u, v, w)}$ JJ′ = 1.

9. Let X,Y, Z be vector space, A∈L (X, Y) and B∈L (X, Y). BA is linear, then A^{-1} is :

(a) Invertible (b) Non-invertible

(c) Non-linear (d) Linear and Invertible

10. If series Σa_n, Σb_n, Σc_n converge to A, B, C respectively and if $C_n = a_0 b_n + a_1 b_{n-1} + \ldots\ldots + a_n b_0$, then c is equal to :

(a) $(AB)^{-1}$ (b) A+ B

(c) AB (d) $A^{-1} + B^{-1}$

SECTION 'B' 4 × 5 = 20

(Short Answer Type Questions)

Note : *Answer the following questions.*

1. State and prove the Cauchy's criterion for uniform convergence.

Or

Prove that the series :

$\frac{1}{a} - \frac{2a}{a^2-1}$ cos x + $\frac{2a}{a^2-2^2}$ cos $2x$ ……… is uniformly convergent in any finite interval.

2. Find the radius of convergence of the power series $\sum_{n=0}^{\infty} \frac{Z^n}{\sqrt{n+1}}$.

Or

If the two real power series $\Sigma a_n x^n$ and $\Sigma b_n x^n$ have radius of convergence R > 0 and converges to the same function in (– R, R), then the two series are identical.

3. Let Ω be the set of all invertible linear operators on R^n,, if A∈Ω, B∈L(R^n), and $\| B - A \| \| A^{-1} \| < 1$.

Or

Let f maps a convex open set E⊂R^n into R^m , f be differentiable in E and there be a real number M such that' $\| f'(x) \| \le M$ for every $x \in$ E. Then

$$|f(b) - f(a)| \le M\, |b - a| \text{ for all } a \in E,\ b \in E.$$

4. Find the rectangular parallelopiped of surface area a^2 and maximum volume.

Or

Find the largest and smallest distance from (0, 0, 0) to the ellipsoid :

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1,\ 0 < a < b < c.$$

5. Let K be a compact subset of R^n and $\{V_\alpha\}$ be an open cover of K. Then there exist functions $\psi_s, \ldots \psi_s \in \xi(R^n)$ such that

(a) $0 \le \psi_i \le 1$ for $1 \le i \le S$

(b) Each ψ_i has its support in some V_α, and

(c) $\psi_1(x) + \psi_2(x) + \ldots\ldots + \psi_s(x) = 1$ for every $x \in$K.

Or

Suppose T is a G-mapping of an open set E⊂R^n into an open set V⊂R^m, S is a G-mapping on V into an open set W⊂R^p and W is a k-form in W, so that W_S is k-form in V and both $(W_S)_T$ and W_{ST} are k-form in E, where ST is defined by ST(x) = S(T(x)). Thus

$$(W_S)_T = W_{ST.}$$