G-1/132/21 **Roll No……………..**

M.Sc. I Semester Examination, April-2021

MATHEMATICS

Paper III

(Topology-I)

Time : 3 Hours] [Maximum Marks : 80

Note : *All questions are compulsory. Question Paper comprises of 3 sections.* ***Section A*** *is objective type/Multiple Choice questions with no internal choice.* ***Section B*** *is short answer type with internal choice.* ***Section C*** *is long answer type with internal choice.*

SECTION 'A'

(Objective Type Questions)

Choose the correct answer : **1 × 10 = 10**

1. For X = $\{a, b, c, d\}$, which of the following is not a topology on X :

(a) T = $\{\phi, \{a\}, \{b\}, \{a, b\}, X\}$

(b) T = $\{\phi, \{b\}, \{c\}, \{b, c\}, \{c, d\}, \{b, c, d\}, X\}$

(c) T = $\{\phi, \{b\}, \{c\}, \{a, c\}, \{b, c\}, \{b, c, d\}, X\}$

(d) T = $\{\phi, \{a\}, X\}$

2. "The cardinality of the set of all subsets of any set is strictly greater than the cardinality of the set", is the statement of :

(a) Well ordering principle

(b) Zorn's Lemma

(c) Cantor's theorem

(d) None of these

P.T.O.

3. Which of the following statement is false ?

(a) Every regular Lindelöff space is normal

(b) Every regular separable space is normal

(c) Product of two normal spaces is normal

(d) Every regular second countable space is normal

4. A topological space X is Housdorff iff :

(a) X is clopen

(b) Every sequence in X has a unique limit

(c) X is closed and bounded

(d) None of these

5. The closure of connected subset is not connected.

(a) True (b) False

6. Which of the following statement is not correct ?

(a) $A \subseteq B \Leftrightarrow \overline{A} \subseteq \overline{B}$ (b) $A \subseteq B \Rightarrow C(A) \subseteq C(B)$

(c) $A^{\circ} = A - A^{b}$ (d) $\overline{A} = A \cup A'$

7. Let X and Y be topological spaces and let $f : X \to Y$ be a function, then which of the following statement is not correct ?

(a) f is continuous iff $f^{-1}(F)$ is closed in X, whenever F is closed in Y

(b) f is continuous iff $f(S)$ is open in Y, whenever S is subbasis open set in Y

(c) f is continuous iff for each $A \subset X$, $f\left(\overline{A}\right) \subset \overline{f(A)}$

(d) f is continuous iff for each $B \subset Y$, $\overline{f^{-1}(B)} \subset f^{-1}\left(\overline{B}\right)$

3. Prove that the space X is a T_1-space iff for any $x \in X$. the singleton set $\{x\}$ is closed.

Or

State and prove Tietze extension theorem.

4. Prove that a topological space X is compact iff every family of closed subsets of X, which has the finite Intersection property has non-empty intersection.

Or

Explain regular space and Lindelöff space and prove that every Lindelöff space is normal.

5. Define components and prove that every component of a topological space is closed but not necessarily open.

Or

Prove that :

(a) Any two distinct components are mutually disjoint

(b) Every non-empty connected subset is contained in unique component

(c) Every space is a disjoint union of its components.

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8. Every metric space is Housdorff.

(a) True (b) False

9. Every compact, Housdorff topological space is locally compact.

(a) True (b) False

10. Which of the following statement is not correct ?

(a) connectedness is a hereditary property

(b) connectedness is a topological property

(c) If C is a dense subset of space X and if C is connected, then X is connected

(d) None of these

SECTION 'B' **4 × 5 = 20**

(Short Answer Type Questions)

Note : *Answer the following questions.*

1. If a space is second countable, then every open cover of it has a countable subcover.

Or

Let (X,T) be a topological space and $A \subseteq X$. Then prove that in*t*(A) is the union of all open sets contained in A.

2. Let X and Y be toplogical spaces and Let $f : X \to Y$ be a function. Prove that f is continuous iff for each $B \subset Y$. $\overline{f^{-1}(B)} \subset f^{-1}\left(\overline{B}\right)$.

Or

Let A and B be subsets of a topological space (X,T), then prove that :

$$\overline{A \cup B} = \overline{A} \cup \overline{B}.$$

3. Prove that regularity is hereditary property.

Or

Prove that $T_2 \Rightarrow T_1$.

4. A metric space X is sequentially compact iff it has Bolzano-Weierstrass property.

Or

Let f be mapping of a locally compact space X onto a Housdorff space Y. If f is both continuous and open, then prove that Y is also locally compact.

5. Prove that every component of a locally connected space is an open set.

Or

If two topological spaces X and Y are homeomorphic, then prove that X is connected iff Y is connected.

SECTION'C' **10 × 5 = 50**

(Long Answer Type Questions)

Note : *Answer the following questions.*

1. Let X be a set. T a topology on X and S a family of subsets of X. Then S is a subbase for T iff S generates T.

Or

Let (X,T) be a topology space and Let $A \subseteq X$. If A′ is the set of all limit points, then the closure of A is $\overline{A} = A \cup A'$.

2. Every second countable space is separable.

Or

Let $f : (X,T) \to (Y, U)$ be a function. Then prove that f is continuous iff for any closed subset F of Y, $f^{-1}(F)$ is closed in X.