G-1/133/21 **Roll No……………..**

M.Sc. I Semester Examination, April-2021

MATHEMATICS

Paper IV

(Advanced Complex Analysis-II)

Time : 3 Hours] [Maximum Marks : 80

Note : *All questions are compulsory. Question Paper comprises of 3 sections.* ***Section A*** *is objective type/Multiple Choice questions with no internal choice.* ***Section B*** *is short answer type with internal choice.* ***Section C*** *is long answer type with internal choice.*

SECTION 'A'

(Objective Type Questions)

Choose the correct answer : **1 × 10 = 10**

1. In Cauchy's Goursat theorem $f(z)$ is :

(a) Continuous function (b) Constant function

(c) Entire function (d) Analytic function

2. The value of the integral $\int_C z\,dz$, where c is unit circle $|z| = 1$ is :

(a) 0 (b) 1

(c) πi (d) $2\pi i$

3. Total number of the roots of the equation $z^8 - 4z^5 + z^2 - 1 = 0$, that lie inside unit circle $|z| = 1$ is :

(a) 2 (b) 3

(c) 4 (d) 5

4. The residue of the function $f(z) = \dfrac{1}{z^2 + a^2}$ at $z = ia$ is :

(a) $\dfrac{1}{2a}$ (b) $\dfrac{-1}{2a}$

(c) $\dfrac{-1}{2a}$ (d) None of these

5. "Let $f(z)$ be analytic inside and on a closed contour C and let $f(z) \neq 0$ inside C. Then $|f(z)|$ attains its minimum value on C and not inside C" is the statement of :

(a) Minimum modulus principle

(b) Maximum modulus principle

(c) Argument principle

(d) Rouche's theorem

6. If $0 \leq \theta \leq \dfrac{\pi}{2}$, then $\dfrac{2\theta}{\pi} \leq \sin\theta \leq \theta$ is called :

(a) Jorden's Lemma (b) Jorden's Inequality

(c) Schwarz Inequality (d) Schwarz Lemma

7. Bilinear transformation is also called :

(a) Linear transformation

(b) Elementary transformation

(c) Inverse transformation

(d) Mobius transformation

Or

Show that the equation $z^4 + 4(1 + i)z + 1 = 0$ has one root in each quadrant.

3. Apply the calculus of residue, to prove that :

$$\int_0^\infty \frac{dx}{(1+x^2)^2} = \frac{\pi}{4}.$$

Or

Apply the calculus of residue to prove that :

$$\int_0^\infty \frac{\sin \pi x}{x(1-x^2)} dx = \pi.$$

4. Prove that, every bilinear transformation maps circles or straight lines into circles or straight lines.

Or

Show that the transformation $(w + 1)^2 = \dfrac{4}{z}$, the unit circle in w-plane corresponds to a parabola in z-plane and inside of the circle to the outside of the parabola.

5. State and prove Riemann mapping theorem.

Or

State and prove open mapping theorem.

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8. An analytic function $f(z)$ in a domain D is said to be conformal is inside D :

(a) $f(z) = 0$ (b) $f'(z) = 0$

(c) $f'(z) \neq 0$ (d) $f(z) = 0$

9. The closure of a totally bounded subset is :

(a) not bounded (b) totally bounded

(c) locally unbounded (d) none of these

10. In Montel's theorem $F \subseteq H(D)$ is normal if and only if F is :

(a) normal (b) continuous

(c) equicontinuous (d) locally bounded

SECTION 'B' **5 × 4 = 20**

(Short Answer Type Questions)

Note : *Answer the following questions.*

1. If C is closed contour containing the origin inside it, prove that :

$$\frac{a^n}{\lfloor n} = \frac{1}{2\pi i}\int_C \frac{e^{az}}{z^{n+1}}\,dz$$

Or

State and prove Liouville's theorem.

2. If $a > e$, use Rouche's theorem to prove that the equation $e^z = az^n$ has n roots inside the circle $|z|=1$.

Or

State and prove Schewarz Lemma.

3. State and prove Cauchy's residue theorem.

Or

Apply calculus of residues to prove that :

$$\int_0^{2\pi} \frac{\cos 2\theta}{5+4\cos\theta}\,d\theta = \frac{\pi}{6}\cdot$$

4. Prove that cross-ratios are invariant under a bilinear transformation.

Or

Show that the transformation $W = \dfrac{2z+3}{z-4}$ maps the circle $x^2 + y^2 - 4x = 0$ onto the straight line $4u + 3 = 0$ and explain why the curve obtained is not a circle.

5. State and prove Hurwitz's theorem for the space of analytic functions.

Or

Prorve that $C(G, \Omega)$ is a complete metric space.

SECTION'C' **10 × 5 = 50**

(Long Answer Type Questions)

Note : *Answer the following questions.*

1. Prove that, if $f(z)$ be analytic within and on the boundary C of a simply connected region D and let a be any point within C. Then derivatives of all orders are analytic and given by :

$$f^{(n)}(a) = \frac{\lfloor n}{2\pi i}\int_C \frac{f(z)\,dz}{(z-a)^{n+1}}$$

Or

State and prove Laurent's Theorem.

2. State and prove Rouche's theorem.