G-1/134/21 **Roll No.…………...**

M.Sc. I Semester Examination, April-2021

MATHEMATICS

Paper V

(Advanced Discrete Mathematics-I)

Time : 3 Hours] [Maximum Marks : 80

Note : *All questions are compulsory. Question Paper comprises of 3 sections.* ***Section A*** *is objective type/Multiple Choice questions with no internal choice.* ***Section B*** *is short answer type with internal choice.* ***Section C*** *is long answer type with internal choice.*

SECTION 'A'

(Objective Type Questions)

Choose the correct answer : **1 × 10 = 10**

1. Which is molecular statement ?

(a) This is my book (b) Delhi is capital of India

(c) MP is in India and Bhopal is capital of M.P.

(d) None of the above

2. Which is contradiction ?

(a) $\sim[p \wedge (\sim p)]$ (b) $p \Leftrightarrow (\sim p)$

(c) $\sim p \vee q$ (d) $p \Rightarrow (\sim p) \Leftrightarrow (\sim p)$

3. Which is cyclic monoid ?

(a) $M = \{1, -1, i, -i\}$ (b) $M = \{x + iy, -2, 1, 0\}$

(c) $M = \left\{\frac{1}{2}, -\frac{1}{2}, \frac{1}{3}, \frac{i}{3}\right\}$ (d) None of the above

4. Which one is concatenation for $a, b \in V^*$?

(a) $a \oplus b \in V^*$ (b) $a - b \in V^*$

(c) $a \div b \in V^*$ (d) $a.b \in V^*$

5. An element $b \in L$ is called complement of an element $a \in L$ if :

(a) $a * b = 1$ (b) $a \oplus b = 0$

(c) $a * b = 0$ (d) $b * 1 = 1$

6. For $a, b \in L$, $a \oplus a * b = a$ is called :

(a) Idempotent law (b) Absorption law

(c) Isotonicity law (d) Bounded law

7. No. of distinct terms in disjunctive normal form of n variables are :

(a) 2^n (b) 2^{n-1}

(c) $2^n - 1$ (d) $\frac{1}{2}.2^n$

8. Value of $a.b + a.b' + a'.b + a'.b'$ for $a, b \in B$ is :

(a) 0 (b) a

(c) b (d) 1

9. Value of $[[(a'.b')' + c] + (a' + c')]'$ for $a, b, c \in B$ is :

(a) 0 (b) a

(c) b (d) 1

10. The value of regular expression is :

(a) grammar (b) string

(c) language (d) length

Or

Prove that for Boolean algebra :

(a) $a.b + [(a + b').b]' = 1$,

(b) $[a + (a' + b)'].[(a + (b'.c)'] = a \ \forall a, b, c \in B$.

5. Define formula and rank for Polish Notation. Translate infix string.

$$(a + b \uparrow c \uparrow d) * \left(e + \frac{f}{d}\right)$$ to Polish Notation with rank

Or

Define Regular sets. Show that $L = \{a^n b^n : n \geq 1\}$ is not regular.

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SECTION 'B' 5 × 4 = 20

(Short Answer Type Questions)

Note : *Answer the following questions.*

1. By truth table the following is Tautology or contradiction :

$$(p \Leftrightarrow q \wedge r) \Rightarrow (\sim r \Rightarrow \sim p)$$

Or

Suppose (S, ∗), (T, Δ) and (V,∇) are semigroups $f : S \to T$ and $g : T \to V$ be semigroup homomorphism. Then $gof : S \to V$ is a semigroup homomorphism from (S, ∗) to (V, ∇).

2. State and prove Basic Homomorphism theorem.

Or

Let S be a set containing n elements, let X′ denote the free semigroup generated by X and let (S, ∗) be any other semigroup generated by n generators, then there exists a homomorphism $f : X' \to S$.

3. State and prove isotonicity property of lattice.

Or

The following Boolean switching function, replace if by simpler one :

$$F(x, y, z) = x.z + [y.(y' + z). (x' + x.z').$$

4. Change the following into conjunctive normal form :

$$E(x_1, x_2, x_3) = x_1.x_2 + x_1.x_3 + x_2'.x_3$$

Or

Use K-map to find minimal sum of product

$$\Sigma f(a, b, c) = \Sigma (0, 1, 4, 6)$$

5. Show that rank of any well-formed Polish Formula is one.

Or

State and prove Kleen's Theorem.

SECTION'C' 10 × 5 = 50

(Long Answer Type Questions)

Note : *Answer the following questions.*

1. Define converse, inverse and contrapositive for any statement find converse, inverse and contrapositive of the following, when sun rises then it is morning.

Or

Let S be any nonempty set and ρ(S) be its power set. Define power set. The algebriac system (ρ(S),U) and (ρ(S),U) are monoids with identities ϕ and S respectively. Given example.

2. Let X be the set of rational numbers and ∗ be the operation on x defined by $a * b = a + b - ab$, show that (X, ∗) be a commutative semigroup.

Or

Let g be a homomorphism from a semigroup (S, ∗) to semigroup (T, ⊕). If H be a subsemigroup of (S, ∗), then $g(H) = \{t \in T : t = g(h) : h \in H\}$ is a subgroup of (T, ⊕) : Define semigroup homomorphism also.

3. State and prove Distributive inequality for lattice.

Or

State and prove Bool's expansion theorem.

4. Replace the following switching circuit by a simpler one.