G-1/139/21 **Roll No……………..**

M.Sc. I Semester Examination, April-2021

PHYSICS

Paper I

(Mathematical Physics)

Time : 3 Hours] [Maximum Marks : 80

Note : *All questions are compulsory. Question Paper comprises of 3 sections.* ***Section A*** *is objective type/Multiple Choice questions with no internal choice.* ***Section B*** *is short answer type with internal choice.* ***Section C*** *is long answer type with internal choice.*

SECTION 'A'

(Objective Type Questions)

Answer the following questions. **1 × 8 = 8**

1. The condition for orthogonal matrices is given by …………

2. The condition for eigen values for matrices is given by ………

3. Write the relation between $P_n(x)$ and $P_n(-x)$.

4. Write the value of $J_{1/2}(x)$.

5. Write the coefficients of Fourier series.

6. Write the value of L{cos at} [Laplace transform of cos (at)].

7. Write Cauchy integral formula.

8. Give the Cauchy-Riemann condition.

SECTION 'B' **6 × 4 = 24**

(Short Answer Type Questions)

Note : *Answer the following questions.*

P.T.O.

Unit I

1. Diagonalize the matrix :

(i) $\begin{bmatrix} \frac{4}{3} & \sqrt{2}/3 \\ \sqrt{2}/3 & 5/3 \end{bmatrix}$

(ii) $\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Or

Explain orthogonal and unitary matrices with example.

Unit II

2. Show that Rodrigue formula for Legendre polynomial is given by :

$$P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n}(x^2 - 1)^n$$

Or

Solve the Bessel differential equation :

$$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 - n^2)y = 0$$

Unit III

3. Give the necessary and sufficient conditions for function to be analytic.

Or

State and prove Cauchy's integral formula.

Or

Apply calculus of residues to prove that :

(i) $\int_0^\infty \frac{x \sin bx}{x^2 + a^2}\, dx = \frac{1}{2}\pi e^{-a} (a > 0)$,

(ii) $\int_0^\infty \frac{x \cos bx}{x^2 + a^2}\, dx = 0$.

Unit IV

4. Give the analytic of square wave in terms of Fourier components.

Or

(a) Find the inverse Laplace transform of :

(i) $\frac{1}{(S+a)(S+b)}$,

(ii) $\frac{S^2}{(S^2 + a^2)^2}$.

(b) Explain the following :

(i) Dirac Delta function,

(ii) Fourier transform and Fourier integral.

✪ ✪ ✪ ✪ ✪ c ✪ ✪ ✪ ✪ ✪

Unit IV

4. Find the Laplace transforms of :

(i) sinh at (ii) cosh at

(iii) sin at (iv) cos at

Or

Evaluate the coefficients of Fourier series.

SECTION'C' **12 × 4 = 48**

(Long Answer Type Questions)

Note : *Answer the following questions.*

Unit I

1. Find the eigen values and normalized eigen vectors of the following matrices :

(i) $\begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$

(ii) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$

Or

Find the inverse of matrices :

(i) $\begin{bmatrix} 1 & -1 & 3 \\ -1 & 1 & 2 \\ 3 & 2 & -1 \end{bmatrix}$

(ii) $\frac{1}{9}\begin{bmatrix} -8 & 1 & 4 \\ 4 & 4 & 7 \\ 1 & -8 & 4 \end{bmatrix}$

Unit II

2. Solve the Hermite differential equation $\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + xy = 0.$

Give the values of Harmite polynomial $H_0(x)$, $H_1(x)$, $H_2(x)$ and $H_3(x)$. Show that :

$$e^{2zx-z^2} = \sum_{n=0}^{\infty} \frac{H_n(x)}{n!} z^n .$$

Or

(a) Prove that for Bessel polynomial

$$\int_0^1 J_n(\alpha x)\, J_n(\beta x) \times dx = \frac{1}{2} J_{n+1}^2(\alpha)\delta_{\alpha\beta}$$

(b) Show that :

$$J_{1/2}(x) = \sqrt{\frac{2}{\pi x}} \sin x$$

$$J_{-1/2}(x) = \sqrt{\frac{2}{\pi x}} \cos x$$

Unit III

3. Prove that contour integration method :

(i) $\int_0^{\pi} \frac{a\, d\theta}{a^2 + \sin^2\theta} = \frac{\pi}{\sqrt{1+a^2}}, a > 0,$

(ii) $\int_0^{2\pi} \frac{d\theta}{1+\sin^2\theta} = 1 + \sqrt{2}.$