G-2/232/21 Roll No. ....

M.Sc. II Semester Examination, 2021

MATHEMATICS

Paper III

(General and Algebraic Topology)

Time : 3 Hours] [Max. Marks : 80

Note : *All questions are compulsory. Question Paper comprises of 3 sections. Section A is objective type/multiple choice questions with no internal choice. Section B is short answer type with internal choice. Section C is long answer type with internal choice.*

SECTION A **1×10=10**

(Objective Type Questions)

Note : *Choose the one correct answer :*

1. Let X_1, X_2 be two topological spaces and let X be the product space. Let P be the projection, defined by $P_1 (x_1, x_2) = x_1$ and $P_2 (x_1, x_2) = x_2$ which of the following is a wrong statement :

(a) P is onto (b) P is continuous

(c) P is open (d) P is closed

2. The diagonal map is :

(a) Continuous (b) One-one

(c) Many one (d) One-one onto

3. Every Tychonoff cube is :

(a) Regular (b) T_0-space

(c) Normal space (d) T_2-space

4. The space (R, U) is :

(a) Metrizable (b) Continuous

(c) T_4-space (d) T_2-space

5. Every metrizable space is :

(a) Countable (b) First countable

(c) Second countable (d) None of these

6. The Unit interval [0, 1] is :

(a) metrizable (b) not metrizable

(c) compact (d) both (a) and (c)

7. A topological space (X, T) is Hausdorff iff net in X can converge to :

(a) many points (b) at least two points

(c) a set (d) unique point

8. If $\{f_a : a \in A\}$ is an ultranet in X and g a mapping of X into Y, then $\{g(f_a) : a \in A\}$ is a :

(a) net (b) ultranet

(c) filter (d) none of these

9. If X is path connected and x_0 and x_1 are two points of X, then $\pi_1 (X, x_0)$ to $\pi_1 (X, x_1)$ is :

(a) Isomorphic (b) Homomorphic

(c) Homotopy (d) None of these

10. The fundamental group of S^I is isomorphic to :

(a) additive group of integer

(b) multiplicative group of integer

(c) additive group modulo (m) of integer

(d) none of these

SECTION B **4 × 5 = 20**

(Short Answer Type Questions)

Note : *Attempt one question from each unit.*

Unit-I

1. Let $T_1 = \{\phi, \{1\}, X_1\}$ be a topology on $X_1 = \{1, 2, 3\}$ and $T_2 = [\phi, X_2, \{a\}, \{b\}, \{a, b\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}]$ be a topology for $X_2 = \{a, b, c, d\}$. Find a base for the product topology.

Or

If (X, T) is the product space of topological spaces (X_1, T_1) and (X_2, T_2), then show that the projection maps are continuous.

Unit-II

2. State and prove Alexander subbase lemma.

Or

State and prove Tychonoff's theorem.

Unit-III

3. Let (X, d) be a compact metric space and (Y, ρ) be a Hausdorff space. Let $f : X \to Y$ be a continuous onto map. Then prove that Y is metrizable.

Or

Show that Discrete topological space is metrizable.

Unit-IV

4. Show that every convergent net in a Hausdorff space X has unique cluster point, which is the unique limit point of the net.

Or

Let (X, T) be a topological space and $A \subset X$. Then prove that a point $x \in X$ belongs to $\bar{A}$ iff these exists a filter base on A converging to x.

Unit-V

5. Define the following :

(i) Homotopy of paths,

(ii) Fundamental group of a topological space.

Or

Define covering space, with example. Show that the map $p : R \to S^1$ given by equation $p(x) = (\cos 2\pi x, \sin 2\pi x)$ is a covering map.

SECTION C **10×5=50**

(Long Answer Type Questions)

Note : *Attempt one question from each unit.*

Unit-I

1. Show that the product space $X = \pi\{X_\alpha : \alpha \in \Lambda\}$ is a T_1 space iff each co-ordinate space is T_1-space.

Or

Prove that the product space $X = \pi\{X_\alpha : \alpha \in \Lambda\}$ is connected iff each co-ordinate space is connected.

Unit-II

2. Let f be a map of a space Y into a product space $X = \pi\{X_\lambda : \lambda \in \Lambda\}$, then prove that f is continuous iff the composition map $\pi_\lambda o f : y \to X_\lambda$ is continuous.

Or

Let y_0 be a fixed element of Y and let $A = X \times \{y_0\}$. Then show that the restriction f_n of π_x to A is a homeomorphism of the subspace A of $X \times Y$ onto X, also show that restriction f_B of π_y to $B = \{x_0\} \times Y$ into Y is a homeomorphism, where $x_0 \in X$.

Unit-III

3. Show that a space X is metrizable if and only if X is regular and has a basis that is countably locally finite.

Or

Prove that a space X is metrizable if and only if it is a paracompact Hausdorff space that is locally metrizable.

Unit-IV

4. Prove that a topological space (X, T) is compact if and only if each not in X has a cluster point.

Or

Show that a filter F on a set X is an ultrafilter iff F contains all those subsets of X which intersect every member of F.

Unit-V

5. Define contractible space and prove the following :

(i) Space X is contractible if and only if for any space Y, any two continuous maps $f : Y \to X$ and $g : Y \to X$ are homotopic.

(ii) Space X is contractible iff $f : X \to Y$ is homotopically equivalent to a one point space.

Or

State and prove the fundamental theorem of Algebra.

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