G-2/233/21 Roll No. ....

M.Sc. II Semester Examination, 2021

MATHEMATICS

Paper IV

(Advanced Complex Analysis-II)

Time : 3 Hours] [Max. Marks : 80

Note : *All questions are compulsory. Question Paper comprises of 3 sections. Section A is objective type/multiple choice questions with no internal choice. Section B is short answer type with internal choice. Section C is long answer type with internal choice.*

SECTION A 1×10=10

(Objective Type Questions)

1. Define Riemann's Functional Equation.
2. Write Legendre's duplication formula.
3. Define Schwarz's reflexion principle.
4. Define complete analytic function.
5. Define Green's function.
6. Define Hormonic function.
7. Write Borel theorem.
8. Define exponent of convergence of sequence $\{z_n\}$.
9. Define univalent functions.
10. Give statement of the Great Picard theorem.

P.T.O.

SECTION B 5×4=20

(Short Answer Type Questions)

Note : *Attempt one question from each unit.*

Unit-I

1. Prove that $T(z) = \lim_{n\to\infty} \frac{n/n^2}{z(z+1).....(z+n)}$.

Or

If $R_e\, z > 1$, then prove that $G(z) = \prod_{n=1}^{\infty}\left(\frac{1}{1-P_n^{-z}}\right)$ where $\{P_n\}$ is the sequence of prime numbers.

Unit-II

2. Show that the series :

(i) $\sum_{n=0}^{\infty} \frac{2^n}{z^n+1}$ and (ii) $\sum_{n=1}^{\infty} \frac{(z-i)^n}{(z-i)^{n+1}}$

are analytical continuation of each other.

Or

Show that there cannot be more than on continuation of an analytic function $f(z)$ into the same domain.

Unit-III

3. State and prove mean value theorem.

Or

Prove that the Poisson kernel $P_r(\theta)$ satisfies the following property : $\frac{1}{2\pi}\int_{-\pi}^{\pi} P_r(\theta)\, d\theta = 1$.

Unit-IV

4. Write the entire function sin z and cos z as canonical product.

Or

Find the order of function cos z.

Unit-V

5. Let f be an analytic function in the disc $B(a, r)$ such that $|f'(z) - f'(a)| < |f'(a)| \;\forall\; z \in B(a, r)$. Then show that f is one-one.

Or

Let f be an entire function that omits two values. Then prove that f is a constant.

SECTION C **10×5=50**

(Long Answer Type Questions)

Note : *Attempt one question from each unit.*

Unit-I

1. Let K be a compact subset of the region G_1. Then prove that there are straight line segments γ_1,

γ_2, ..., γ_n in G-K such that for every function f in $H(G)$

$$f(z) = \sum_{k=1}^{n} \frac{1}{2\pi i}\int_{\gamma_k} \frac{f(w)}{w - z}\, dW \;\forall\; z \in K.$$

The line segment from a finite number of closed polygons.

Or

State and prove Weierstrass Factorization theorem.

Unit-II

2. Show that the function $f_1(z) = 1 + z + z^2 + z^3 + ...$ can be obtained outside the circle of convergence of the power series.

Or

Prove that unit circle $|z| = 1$ is natural boundary of the function $f(z) = \sum_{n=0}^{\infty} z^{n!}$.

Unit-III

3. Let G be a region and let $a \in \partial_\infty G$ such that there is a barrier for G at a. If $f : \partial_\infty G \to R$ is continuous and u is the person function associated with f, then $\lim_{z \to a} u(z) = f(a)$.

Or

Let G be a region and let u and v be two continuous real-valued function of G that have *MVP*. If for each point a in the extended bounded $\partial_\infty G$ $\limsup_{z \to a} u(z) \le \liminf_{z \to a} v(z)$

Then prove that either $u(z) < v(z)$ $\forall$ $z \in G$ or $u = v$.

Unit-IV

4. Let $f(z)$ be analytic in closed disc $|z| \le R$. Assume that $f(0) \ne 0$ and *no* zeros of $f(z)$ lie on $|z| = R$. If $z_1, z_2, \ldots\ldots, z_n$ are the zeroes of $f(z)$ in the open disc $|z| < R$, each repeated as often as its multiplicity, then

$$\log|f(0)| = -\sum_{i=1}^{n} \log\left(\frac{R}{|z_i|}\right) + \frac{1}{2\pi}\int_0^{2\pi} \log|f(Re^{i\phi})|\, d\phi.$$

Or

State and prove Hadmard's Factorization Theorem.

Unit-V

5. State and prove Montel Caratheodory theorem.

Or

Let, $f \in \varphi$ and $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$, then

(a) $|a_2| \le 2$ and (b) $f(v) \supset D\left(0, \frac{1}{4}\right)$

then second assertion say's that $f(v)$ contains all W with $|W| < \frac{1}{4}$.

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