G-3/330/21 Roll No................

M.Sc. III Semester Examination, April-2021

MATHEMATICS

Paper I

(Integration Theory and Functional Analysis-I)

Time : 3 Hours] [Maximum Marks : 80

Note : *All questions are compulsory. Question Paper comprises of 3 sections.* **Section A** *is objective type/Multiple Choice questions with no internal choice.* **Section B** *is short answer type with internal choice.* **Section C** *is long answer type with internal choice.*

SECTION 'A' **1×10=10**

(Objective Type/Multiple Type Questions)

Choose the correct answer :

1. What is the difference between a null set and a set of measure zero ?

2. Define comple measure space.

3. What do you mean by distribution function ?

4. Define simple function.

P. T. O.

5. Define regular measure.

6. Give an example of Baire and a Borel set.

7. Define bicontinuous function.

8. Define reflexive space.

9. What is difference between weak and weak* convergence.

10. Define dual spaces.

SECTION 'B' **4×5=20**

(Short Answer Type Questions)

Note : *Answer the following questions in 250 words.*

1. Show that the union of a countable collection of negative sets is negative.

Or

Set (X, B, μ) as a finite measure space and g an integrable function such that for some constant M,

$$| g \, \phi \, d\mu | < M \, \| \phi \|_p$$

for all simple functions ϕ. Then $g \in L^q$.

2. Set $\{(A_i X B_i)\}$ be a countable disjoint collection of measurable rectangles whose union is a measurable rectongle A × B. Then

$$\lambda \, (A \times B) \sum_{i=i}^{\infty} = \lambda \, (A_i \times B_i)$$

Or

Set y be a point of Y and E a set in Rσs. Then E_y is a measurable rectangle of X.

3. Show that every compact Baire set is a G_8.

Or

Every σ- bounded open set is a Borel set.

4. Define equivalent norm. Show that on a finite dimensional linear space X, any norm $\| \cdot \|_2$ is equivalent to any other norm $\| \cdot \|_2$.

Or

Show that the function space C [a, b] is a Banach space.

5. (a) Show that B (X, Y) is Banach space if Y is a Banach space.

Or

(b) Define weak* convergence and show that weak limit of a sequence is unique.

SECTION 'C' **10×5=50**

(Long Answer Type Questions)

Note : *Answer the following questions in 500 words.*

1. State and prove Radon Nikodym theorem.

Or

State and prove Riesz representation theorem.

2. Define measurable subsets. Let E be a measurable subset of X×Y such that μ × υ (E) is finite. Then for almost all x the set E_x is a measurable subset of Y. The function g defined by

$$g (x) = \upsilon (E_x)$$

is a measurable function defined for allmost all x and

$$\int g \, d\mu = \mu \times \upsilon (E).$$

Or

State and prove Fubixi theorem.

3. Set μ be a Baire measure on a locally compact space X and E a σ-bounded Baire set in X. Then for ε > 0;

(i) There is a σ- compact open set O with

$E \subset O$ and $\mu (O \sim E) < \varepsilon$

(ii) $\mu E = \sup \{\mu K : K \subset E, K \text{ a compact } G_8\}$.

Or

State and prove Riesz-Markov theorem.

4. Let X be a normed linear space. The closed unit ball

$$B = \{x \in X : \| x \| <| 1\}$$

in X is compact if any only if X is finite dimensional.

Or

Define Quotient space. Set M be a closed linear subspace of X. Then X/M is Banach space if X is Banach.

5. Show that C* is isometrically isomorphic to l_1.

Or

What do you mean by strong and weak convergence ? Show that if X is finite dimensional then W each convergenece implies strong convergence.

✪ ✪ ✪ ✪ ✪ c ✪ ✪ ✪ ✪ ✪