G-3/331/21 **Roll No.……...**

M.Sc. III Semester Examination, April-2021

MATHEMATICS

Paper II

(Partial Differential Equation and Mechanics-I)

Time : 3 Hours] [Maximum Marks : 80

Note : *All questions are compulsory. Question Paper comprises of 3 sections.* ***Section A*** *is objective type/Multiple Choice questions with no internal choice.* ***Section B*** *is short answer type with internal choice.* ***Section C*** *is long answer type with internal choice.*

SECTION 'A'

(Objective Type Questions)

Choose the correct answer : **1 × 10 = 10**

1. Let $u : R^n \to R$ is harmonic and bounded. Then u is constant is called :

(a) Laplace Theorem (b) Poisson Theorem

(c) Liouville Theorem (d) Mean Value Theorem

2. Which is Poisson Kernel for R^4_+ ?

(a) $2x_n/n.\alpha(n)|x-y|^n$ (b) $\dfrac{x_n^2}{n.\alpha(n)|x-y|^n}$

(c) $nx_n/\alpha(n)|x-y|^2$ (d) $\dfrac{x_n}{\alpha(n)}|x-y|^4$

3. Which is green function for half space R^4_+?

(a) $\phi(Y-X)-\phi^x(Y-\overline{X})$ (b) $\phi(Y-X)-\phi(Y-\overline{X})$

(c) $\phi(Y-X)-\phi^x(Y)$ (d) $\phi(Y-X)-\phi(X)$

P.T.O.

4. Complete integral of equation $x.Du + f(Du) = 4$ is :

(a) $u(x, a) = x/a + f(a)$ (b) $u(x, a) = \frac{a}{x} + f(a)$

(c) $u(x, a) = ax + f'(a)$ (d) $u(x, a) = a.x + f(a)$

5. Relation between H and L is :

(a) $H(P, X) = q(P, X) - PL(q(P - X)X)$

(b) $H(P, X) = P - q.L(q(P, X)X)$

(c) $H(P, X) = Pq(P, X) - L(q(P, X), x)$

(d) $H(P, X) = P - q(P,X) - L(q(PX)X)$

6. Equation $U_t(x, t) + H(Du(x,t)) = 0$ is called :

(a) Hamilton equation

(b) Jacobi equation

(c) Euler-Lagrange equation

(d) Hamilton-Jacobi equation

7. Which is entropy condition :

(a) $F'(u_1) + 6 > F'(u_r) - 6$ (b) $F'(u_t) > 6 > F'(u_r)$

(c) $F(u_l) > 6' > F(u_r)$

(d) $F'(u_l) - 6 < F'(u_r) + 6$

8. Attraction of thin uniform spherical shell at inside the shell is :

(a) 0 (b) γ-mass of shell /OP^2

(c) $\frac{c^2 - a^2}{c + a}$ (d) $\frac{c^2 - a^2}{c - a}$

4. Prove that the attraction of uniform rod of infinite length varies inversely as the distance of the point from the rod.

Or

Prove that potential of a thin uniform spherical shell at an internal point varies directly as the radius of spherical shell.

5. If N be normal the attraction and M be amount of the attracting mass, then show that $\int N ds = -4\pi\gamma M$.

Or

State and prove Poisson equation for normal attraction.

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9. Potential of uniform circular disc at its centre is :

(a) 0 (b) Reciprocal to radius of disc

(c) proportion to radius of disc

(d) proportion of square of its radius (disc)

10. Formula for work done by self attracting system for two mass m_1 and m_2 is :

(a) $\gamma.\frac{m_1 m_2}{r^2}$ (b) $\gamma.\frac{m_1 / m_2}{r^2}$

(c) $\gamma.\frac{m_1 / m_2}{r}$ (d) $\gamma.\frac{m_1 m_2}{r}$

SECTION 'B' **5 × 4 = 20**

(Short Answer Type Questions)

Note : *Answer the following questions in 250 words.*

1. State and prove Harnack's inequality.

Or

Derive the formula for Green's function.

2. Explain strong maximum principle for the heat equation.

Or

State and prove uniqueness for wave equation.

3. State and prove Hopf Lax formula.

Or

State and prove Euler Lagranges' equation.

4. Find the potential of a uniform circular plate at a point on its axis.

Or

Find the attraction of a solid sphere at an external and internal points.

5. A self attracting sphere of uniform density and radius a changes to one of uniform density and radius b, show that the work done by its mutual attractive forces is $\frac{3}{5}\gamma M^2\left(\frac{1}{b}-\frac{1}{a}\right)$ where M is the mass of the sphere.

Or

If S be a closed equipotential surface, of potential V, which includes mass M of the attracting matter and if a thin stratum of attracting matter be placed on S, whose density ρ at any point P is equal to $-\frac{1}{4\pi\gamma}.\frac{dv}{dn}$, where δn is outward normal at P.

SECTION 'C' **10 × 5 = 50**

(Long Answer Type Questions)

Note : *Answer the following questions in 500 words.*

1. State and prove estimates on derivatives for u.

Or

State and prove Poisson formula for half space.

2. Write the solution of wave equation for $n = 1$ and prove it.

Or

State and prove Euler-Poisson-Darboux equation.

3. State and prove Lax Oleinix formula.

Or

State and prove convex dulity of Hamilton and Lagrangian function.