Determine the shortest route and hence shortest distance from city 1 to city 7.

Or

A small project consists of seven activities for which the relevant data are given below :

Activity	Preceding Activity	Activity duration (days)
A	—	4
B	—	7
C	—	6
D	A, B	5
E	A, B	7
F	C, D, E	6
G	C, D, E	5

(i) Draw the network and find the project completion time.

(ii) Calculate total float for each of the activities and highlight the critical path.

(iii) Draw the time scaled diagram.

✪ ✪ ✪ ✪ ✪ c ✪ ✪ ✪ ✪ ✪

G-3/333/21 **Roll No.……...**

M.Sc. III Semester Examination, April-2021

MATHEMATICS

Paper IV

(Operations Research-I)

Time : 3 Hours] [Maximum Marks : 80

Note : *All questions are compulsory. Question Paper comprises of 3 sections.* ***Section A*** *is objective type/Multiple Choice questions with no internal choice.* ***Section B*** *is short answer type with internal choice.* ***Section C*** *is long answer type with internal choice.*

SECTION 'A'
(Objective Type Questions)

Choose the correct answer : **1 × 10 = 10**

1. Decision variables in OR, model are :

(a) controllable (b) uncontrollable

(c) parameters (d) constant

2. Given system of M simulteneous linear equations in n unknowns ($m < n$) the number of basic variable will be :

(a) m (b) n

(c) $n - m$ (d) $n + m$

3. If a dual has an unbounded solution, the primal has :

(a) an unbounded solution

(b) infeasible solution

(c) a feasible solution

(d) none of the above

4. Sensitivity analysis is a technique to :

(a) allocate resources optimality

(b) determine how optional solution to an LPP changes response to problem input

(c) Minimize cost of operations

(d) None of the above

5. Deviational variable in goal programming problem must statisfy following conditions :

(a) $d_i^+ + d_i^- = 0$ (b) $d_i^+ - d_i^- = 0$

(c) $d_i^+ \times d_i^- = 0$ (d) $d_i^+ / d_i^- = 0$

6. Parametric programming is extension of :

(a) sensitivity analysis (b) dual Simplex method

(c) Big-M method (d) all of the above

7. The dummy source or destination in transportation problem (TP) is introduced to :

(a) Prevent solution to become degenerate

(b) to satisfy rim conditions

(c) Ensure that total cost does not exceed a limit

(d) None of the above

8. An assignment problem (AP) is considered as a particular case of T.P. because :

(a) All the rim conditions are 1

(b) All x_{ij} are either 1 or 0

(c) The number of rows equals columns

(d) All of the above

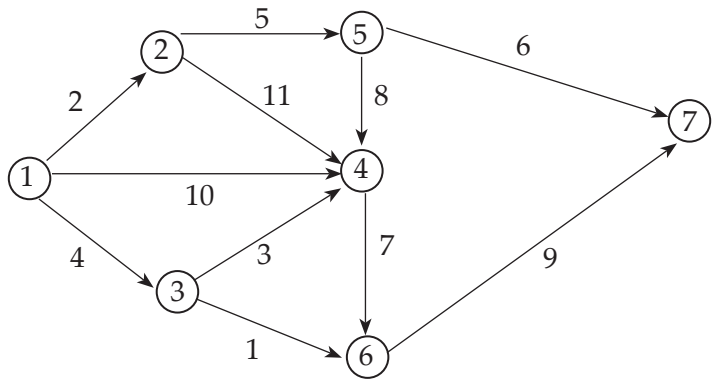
		S_1	S_2	S_3	S_4	Availability
Warehouse	A	5	1	3	3	34
	B	3	3	5	4	15
	C	6	4	4	3	12
	D	4	1	4	2	19
Demand		21	25	17	17	80

Or

Solve the assignment problem represented by the following matrix :

	I	II	III	IV	V	VI
A	9	22	28	11	19	27
B	43	78	72	50	63	48
C	41	28	91	37	45	33
D	74	42	27	49	39	32
E	36	11	57	22	25	18
F	3	56	53	31	17	28

5. The network below gives the permissible routes and their length in miles between stations of city 1 (node 1) and six other cities (node 2–7).



(i) Formulate the underlying L.P.P.

(ii) Write "dual" of the problem.

(iii) Solve the "dual" problem by Simplex method.

3. Using the bounded variable technique, solve the L.P.P. :

$$\text{Maximize } Z = 3x_1 + 5x_2 + 2x_3$$

Subject to constaints :

$$x_1 + 2x_2 + 2x_3 \leq 14$$

$$2x_1 + 4x_2 + 3x_3 \leq 23$$

$$0 \leq x_1 \leq 4$$

$$2 \leq x_2 \leq 5$$

$$0 \leq x_3 \leq 3.$$

Or

Solve the following Goal programming problem using Simplex method :

$$\text{Minimize } Z = P_1 d_1^- + P_2 d_2^- + 2P_2 d_2^- + P_3 d_1^+$$

Subject to constraints

$$10x_1 + 10x_2 + d_1^- - d_1^+ = 400$$

$$x_1 + d_2^- = 40$$

$$x_2 + d_3^- = 30$$

$$x_1, x_2 d_1^+, d_1^-, d_2^- d_3^- \geq 0.$$

4. Obtain an initial BFS to the following transportation problem using

(i) N–W corner rule,

(ii) Vogels Approximation method.

9. A typical application of minimal spanning tree involves :

(a) Construction of paved roads and link several towns

(b) determining the least cost path between two cities

(c) finding the shortest path communication network

(d) All of the above

10. Network problems have advantage in terms of project :

(a) scheduling (b) planning

(c) controlling (d) all of the above

SECTION 'B' **4 × 5 = 20**

(Short Answer Type Questions)

Note : *Answer the following questions.*

1. Obtain all basic solution of the following system of linear equations :

$$x_1 + 2x_2 + x_3 = 4$$

$$2x_1 + x_2 + 5x_3 = 5.$$

Or

Explain principle of duality in LPP.

2. Prove that the dual of the dual is primal.

Or

State general rule for finding a dual LPP, from its primal.

3. What is goal programming ? Clearly state its assumptions.

Or

Explain the basic idea of interior point used in Karmarkar Algorithm.

4. Prove that a necessary and sufficient condition for the existance of feasible solution to the transportation problem is

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j \, .$$

Or

Give mathematical formulation of Assignment problem.

5. What is dummy activity ? Why it is used ?

Or

Define total, free and independent float with example.

SECTION'C' **10 × 5 = 50**

(Long Answer Type Questions)

Note : *Answer the following questions.*

1. Use penaty (Big-M) method to solve L.P.P. :

$$\text{Minimize } Z = 2x_1 + x_2$$

Subject to constraints :

$$3x_1 + x_2 = 3$$

$$4x_1 + 3x_2 \geq 6$$

$$x_1 + 2x_2 \leq 3$$

and $$x_1 \geq 0, x_2 \geq 0.$$

Or

Consider the L.P.P.

$$\text{Maximize } Z = -x_1 + 2x_2 - x_3$$

Subject to constraints :

$$3x_1 + x_2 - x_3 \leq 10$$

$$-x_1 + 4x_2 + x_3 \geq 6$$

$$x_2 + x_3 \leq 4;$$

and $$x_j \geq 0 \text{ for } j = 1, 2, 3.$$

Determine the ranges for discrete changes in the components b_2 and b_3 of the requirement vector so as to maintain the feasibility of the current optimum solution.

2. Use dual Simplex method to solve L.P.P.

$$\text{Maximize } Z = -2x_1 - x_2$$

Subject to constaints :

$$3x_1 + x_2 \geq 3$$

$$4x_1 + 3x_2 \geq 6$$

$$x_1 + 2x_2 \geq 3$$

and $$x_1 \geq 0, x_2 \geq 0.$$

Or

A diet conscious house wife wishes to ensure certain minimum in take of vitamins A, B and C for the family. The minimum daily (quantity) needs of the vitamins A, B and C for the family are respectively 30, 20 and 16 units. For the supply of these minimum vitamin requirements, the housewife relies on two fresh foods. The first one provides 7, 5, 2 units of the three vitamins per gram respectively and second one provides 2, 4, 8 units of the same three vitamins per gram of the food stuff respectively.

The first food stuff costs Rs. 3 per gram and second Rs. 2 per gram. The problem is how many grams of each food stuff should the housewife buy everyday to keep her food big as low as possible ?