G-4/429/21

Roll No. ....

M.Sc. IV Semester Examination, 2021

MATHEMATICS

Paper II

(Partial Differential Equations Mechanics-II)

Time : 3 Hours] [Max. Marks : 80

Note : *All questions are compulsory. Question Paper comprises of 3 sections. Section A is objective type/multiple choice questions with no internal choice. Section B is short answer type with internal choice. Section C is long answer type with internal choice.*

SECTION A 1×10=10

(Objective Type Questions)

Choose the correct answer :

1. The particular solution of the Hamilton-Jacobi Equation, (using separation of variables method) $u_t + H(Du) = 0$ in $R^n \times (0, \infty)$ is :

(a) $u(x, t) = \omega(x) - \mu t + b$

(b) $u(x, t) = \omega(x) + \mu t - b$

(c) $u(x, t) = a \cdot x - H(a)t + b$

(d) $u(x, t) = a \cdot x + H(a)t - b$.

2. THe following statement

"Assume $u \in L^1(R^n) \cap L^2(R^n)$, $\hat{u}, \hat{u} \in L^2(R^n)$

and $\| \hat{u} \|_{L^2(R^n)} = \| \hat{u} \|_{L^2(R^n)} = \| u \|_{L^2(R^n)}$" states :

(a) Plancherel's Theorem

(b) Bessel's Potential

(c) Fundamental solution of heat equation

(d) None of these.

3. Simple pendulum with rigid support is an example of :

(a) Conservative and Bilateral constraint

(b) Rheonomic and Holonomic constraint

(c) Dissipative and Nonholonomic constraint

(d) None of these.

4. Conservation Theorem for Energy is :

(a) $T + V$ = Constant

(b) $T - V$ = Constant

(c) $2T + V$ = Constant

(d) None of these.

5. For dynamical variables $u(p_j, q_j, t)$, $v(p_j, q_j, t)$ and $w(p_j, q_j, t)$, the identity

$[u, [v, w]] + [v, [w, u]] + [w, [u, v]] = 0$

is known as :

(a) Hamilton-Jacobi Identity

(b) Nelson-Jacobi Identity

(c) Jacobi-Poisson Identity

(d) None of these.

6. For Minimum surface of Revolution, the required formula is :

(a) $x = c_1 \cosh\left(\frac{y - c_2}{a}\right)$ (b) $x = c_1 \sinh\left(\frac{y - c_2}{a}\right)$

(c) $x = c_1 \tanh\left(\frac{y - c_2}{a}\right)$ (d) $x = c_1 \coth\left(\frac{y - c_2}{a}\right)$

7. Define any one :

(a) Configuration space (b) Phase space

(c) State space

8. $\delta\int_{t_1}^{t_2} L\,dt = 0$ is known as :

(a) Hamilton's Principle

(b) Principle of Least Action

(c) Jacobi Equation

(d) Hamilton–Jacobi Equation

9. The generating function used in Hamilton-Jacobi equation is :

(a) $F_1(q_i, \theta_j, t)$ (b) $F_2(q_i, p_j, t)$

(c) $F_3(p_i, \theta_i, t)$ (d) $F_4(p_i, p_j, t)$

10. Harmilton-Jacobi equation for a particle of mass m moving in a force field with its potential given by $V = -\frac{K\cos\theta}{r^2}$ is :

(a) $\frac{1}{2^m}\left[\left(\frac{\partial s}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial s}{\partial \theta}\right)^2 + \frac{1}{r^2 \sin^2\theta}\left(\frac{\partial s}{\partial \theta}\right)^2\right] - \frac{K\cos\theta}{r^2} + \frac{\partial s}{\partial t} = 0$

(b) $\left(\frac{\partial s}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial s}{\partial \theta}\right)^2 + \frac{1}{r^2 \sin^2\theta}\left(\frac{\partial s}{\partial \theta}\right)^2 = 0$

(c) $\left(\frac{\partial s}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial s}{\partial \theta}\right)^2 = 0$

(d) None of these.

SECTION B **4×5=20**

(Short Answer Type Questions)

Note : Attempt all questions.

Unit-I

1. Solve the Hamilton-Jacobi equation $u_t + H(Du) = 0$ in $R_n \times (0, \infty)$ using separation of variables method.

Or

Define Fourier Transform on L'.

Unit-II

2. Derive Lagrange's equation from the Lagrangian function $L(q_k, \dot{q}_k, t) = \dot{q}_k q_k - \sqrt{1-\dot{q}_k^2}$.

Or

Find the Lagrangian function for a simple pendulum and obtain the equation of its motion.

Unit-III

3. Prove that $\frac{d}{dt}[u,v] = \left[\frac{du}{dt}, v\right] + \left[u, \frac{dv}{dt}\right]$.

Or

State and prove Poisson's theorem.

Unit-IV

4. Find a complete solution of the Euler-Lagrange equation for $\int_{x_1}^{x_2} \left[y^2 - \left(\frac{dy}{dx}\right)^2 - 2y\cosh x\right] dx$.

Or

State Hamilton's Principle.

Unit-V

5. State Poincare-Cartan Integral Invariant.

Or

State Lee Hwa-chung theorem.

SECTION C **10×5=50**

(Long Answer Type Questions)

Note : Attempt all questions.

Unit-I

1. Define soliton. Find soliton solution of the Korteweg-de Vries equation $u_t + 6uu_x + u_{xxx} = 0$ in $R \times (0, \infty)$.

Or

State and prove Plancherel's theorem.

Unit-II

2. Establish Lagrange's equations of motion of the second kind.

Or

State and prove Routh's equations.

Unit-III

3. Define Geodesics. Find the shortest distance between two points in a plane.

Or

FInd the geodesics on a sphere.

Unit-IV

4. Derive Hamilton's Principle from Newton's equation.

Or

Derive Hamilton's canonical equations using Hamilton's principle.

Unit-V

5. Use Hamilton-Jacobi equation to discuss the motion of a massive particle in a plane under a central force.

Or

Find the relation between Poisson Bracket and Lagrange Bracket.

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