G-3/376/22

Roll No. ....

III Semester Examination, January 2022

M.Sc.

MATHEMATICS

Paper I

(Integration Theory and Functional Analysis-I)

Time : 3 Hours] [Max. Marks : 80

Note : *All questions are compulsory. Question Paper comprises of 3 Sections. Section A is objective type/multiple choice questions with no internal choice. Section B is short answer type with internal choice. Section C is long answer type with internal choice.*

SECTION A **1×10=10**

(Objective Type/Multiple Choice Questions)

Choose the correct answer :

1. Two measure V_1 and V_2 on $(X_1\beta)$ are said to be mutually singular (*ee* $V_1 \perp V_2$) if these are disjoint measurable set A and B with $X = A \cup B$ such that :

(a) $V_1 (A) = V_2 (B) \neq 0$

(b) $V_1 (B) = V_2 (A) = 0$

(c) $V_1 (A) = 0$ but $V_2 (B) \neq 0$

(d) None of the above

2. Let (X, β, μ) be a σ finite measure space and let ν be a measure define on β which is absolutely continuous with respect of μ then there is a non-negative measurable function f such that for each set E in β we have

$$\upsilon E = \int_E f\, du$$

The above statement is :

(a) Hahn Decomposition theorem

(b) Radon Nikodyn theorem

(c) Lebesgue decomposition

(d) Riesz representation theorem

3. The x-cross–section of any measurable subset E of $X \times Y$ is define by :

(a) $Ex = \{x : (x, y) \in E\}$

(b) $Ey = \{y : (x, y) \in E\}$

(c) $Ex = \{y : (x, y) \in E\}$

(d) $Ey = \{x : (x, y) \in E\}$

4. If the derivative of two absolutely continuous function are equivalent then the :

(a) Function differ by a constant

(b) Function differ by a variable

(c) Function is not constant

(d) None of the above

5. A measure μ is called regular measure if :

(a) μ is inner regular

(b) μ is outer regular

(c) μ is both inner and outer regular

(d) None of the above

6. A set E is called a .... set if it can be obtained from closed and open set by using a finite or an enumerable number of union and intersection operation.

(a) Borel set (b) Baire set

(c) Both (a) and (b) (d) None of these

7. Which of the following space is not banach space ?

(a) R^n

(b) C^n

(c) C(X), space of all bounded continuous scalar valued functions defined on X

(d) (10, 1), space of real valued continuous function on [0, 1]

8. Consider the following statements :

(i) Every finite dimensional subspace of normed linear space is closed.

(ii) Let X be normed linear space then X is compact if X is finite dimensional

(a) (i) is true and (ii) is false

(b) (i) is false and (ii) is true

(c) (i) and (ii) both are false

(d) (i) and (ii) both are true

9. Let T : X $\rightarrow$ Y where X and Y are normed linear space such that $\| T_x \| \leq M \| x \| \; \forall \; x \in x, \; \mu > 0$ then :

(a) T is linear transformation

(b) T is bounded

(c) T is continuous

(d) All of the above

10. Let $\{x_n\}$ be a weakly convergent sequence in a normed linear space X *i.e.*, $x_n \xrightarrow{\omega} x$ then :

(a) The sequence $\| x_n \|$ is convergent

(b) The sequence $\| x_n \|$ is unique

(c) The sequence $\| x_n \|$ is bounded

(d) None of the above

SECTION B **4×5=20**

(Short Answer Type Questions)

Note : *Answer the following questions.*

Unit-I

1. Let υ be a signed measure on measurable space (X, β) then there is a positive set A and negative set B such that X = A ∪ B and A ∩ B = ϕ, ν assume almost one of the value + ∞ and – ∞.

Or

Let F be a bounded linear function on L^P (μ) with 1 < P < ∞ then there is a unique element $g \in L^q$ such that $F(f) = \int f \cdot g \, d\mu$ we have $\| F \| = \| g \|_q$.

Unit-II

2. Show that the intersection of two σ-compact is a σ-compact.

Or

Varify Fubini theorem for $\int_0^1 dx \int_0^1 \frac{x^2 - y^2}{x^2 + y^2} dy = \int_0^1 dy \int_0^1 \frac{x^2 - y^2}{x^2 + y^2} dx$.

Unit-III

3. Let μ be a finite measure defined on σ algebra a which contains all the Baire sets of a locally compact space X, if μ is inner regular, show that it is regular.

Or

Let μ be a Baire measure on X then show that there is a unique quasi regular Baire measure $\bar{\mu}$ on X and a unique inner regular Baire measure

$\underline{\mu}$ on X such that $\mu\varepsilon = \bar{\mu}\varepsilon = \underline{\mu}\varepsilon$ for every σ bounded Baire set E.

Unit-IV

4. Prove that every complete subspace of normed linear space is closed .

Or

Prove that the linear space R^n equiped with the norm given by :

$$\| x \|_2 = \left(\sum_{i=1}^{n} | \varepsilon_i |^2 \right)^{1/2} ; \; x = (\varepsilon_1, \varepsilon_2 \ldots\ldots \varepsilon_n) \in R^n$$

is a real Banach space.

Unit-V

5. Prove that the Dual space of $l^p(n)$; $1 < p < \infty$ is $l^p(n)$ where $1 < q < \infty$ and $\frac{1}{p} + \frac{1}{q} = 1$.

Or

Let X and Y be normed linear spaces and T : X → Y be any linear transformation if X is finite dimensional then T is continuous.

SECTION C **10×5=50**

(Long Answer Type Questions)

Note : *Answer the following questions.*

Unit-I

1. Let E be a measurable set such that $0 < \nu E < \infty$ then there is a positive set A contain in E with $\nu A > 0$.

Or

State and prove extension theorem (caratheodory).

Unit-II

2. State and prove Lebesque stieltjes integral.

Or

Define bounded variation. Prove that every absolutely continuous function is a bounded variation.

Unit-III

3. Let μ be a measure defined on σ algebaa a containing the Baire sets. Assume that either μ is quasi regular or μ is inner regular. Then for each $E \in a$ with $\mu(E) < \infty$, there is a Baire set B with $\mu(E \Delta B) = 0$.

Or

Let μ^* be a topologically regular outer measure on X. Then each Borel set is μ^* measurable.

Unit-IV

4. Prove that all norms are equivalent on a finite dimensional space.

Or

State and prove Reize's lemma.

Unit-V

5. Let X and Y be normed space $T : X \to Y$ is compact linear operator suppose that sequence $\{x_n\}$ in X is weakely convergent say $x_n \xrightarrow{\omega} x$ then $\{T(x_n)\}$ is strongly convergent in Y and has the limit $Y = T_x$.

Or

Let X and Y be normed linear spaces, then B(X, Y) the set of all bounded linear transformation from X into Y is a normed linear space. More over if Y is a Banach space, then B(X, Y) is also a Banach space.

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