GO-98-21 **Roll No.……...**

Annual Examination, 2021

B.C.A. III (Old Course)

B.C.A.-301

Paper I

(Calculus and Geometry)

Time : 3 Hours] [Maximum Marks : 50

Note : *Attempt any two parts from each unit. All questions carry equal marks.*

Unit I

1. (a) Let $f(x) = x^2$ and $[0, a]$, $a > 0$, show that $f \in R\,[0, a]$ and $\int_0^a x^2\,dx = \frac{a^3}{3}$.

(b) Let $f \in R\,[a, b]$ and let m, M be bounds of on $[a, b]$. Then $m\ (b - a) \leq \int_a^b f(x)\,dx \leq M(b - a)$, if $b \geq a$, $m\,(b - a) \geq \int_a^b f(x)\,dx \geq M(b - a)$, if $a \geq b$.

(c) Prove that :

$$\left|\int_a^b \frac{\sin x}{x}\,dx\right| \leq 2\left(\frac{1}{a} + \frac{1}{b}\right), b > a > 0.$$

P.T.O.

Unit II

2. (a) Find the maximum or minimum value of the function $x^3y^2(1-x-y)$.

(b) Prove that :

$(x + y + z)^3 - 3(x + y + z) - 24xyz + a^3$ has a minimum at (1, 1, 1) and maximum at (– 1, – 1, – 1).

(c) Find the maximum or minimum value of $u = a^2x^2 + b^2y^2 + c^2z^2$ subject to the conditions $x^2 + y^2 + z^2 = 1$ and $lx + my + nz = 0$.

Unit III

3. (a) Test the convergence of $\int_a^{\infty} \frac{dx}{x\sqrt{1+x^2}}$.

(b) Show that $\int_a^{\pi/2} \log \sin x \, dx$ converges.

(c) Prove that the integral $\int_a^{\infty} \frac{\cos \alpha \, dx - \cos \beta x}{x} dx$ is convergent.

Unit IV

4. (a) Find the equation of the cone whose vertex is (5, 4, 3) and base curve $3x^2 + 2y^2 = 6$, $y + z = 0$.

(b) Find the angle between the lines of section of the plane $3x + y + 5z = 0$ and the cone $6yz - 2zx + 5xy = 0$.

(c) Find the equation of the cylinder whose generators are parallel to the line $\frac{x}{1} = \frac{y}{-2} = \frac{z}{3}$ and the base curve is $x^2 + 2y^2 = 1$, $z = 0$.

Unit V

5. (a) Show that the equations $\frac{l}{r} = 1 + e\cos\theta$ and $\frac{l}{r} = -1 + e\cos\theta$ represent the same conic.

(b) In a conic prove that the sum of the reciprocal of two perpendicular focal chords is constant.

(c) To find the polar equation of a conic whose focus is pole, eccentricity is e and latus-rectum is $2l$.

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