G-1/179/22 Roll No. ....

I Semester Examination, January 2022

M.Sc.

MATHEMATICS

Paper IV

(Advanced Complex Analysis-I)

Time : 3 Hours] [Max. Marks : 80

Note : *All questions are compulsory. Question Paper comprises of 3 Sections. Section A is objective type/multiple choice questions with no internal choice. Section B is short answer type with internal choice. Section C is long answer type with internal choice.*

SECTION A **1×10=10**

(Objective Type/Multiple Choice Questions)

Choose the correct answer :

1. The value of the integral $\int_C \frac{z^2 - z + 1}{z - 1}$, where C is the circle $|z| = \frac{1}{2}$ is :

(a) 1 (b) 0

(c) $\frac{1}{2}$ (d) None of these

2. A function which has no singularities in the finite part of the plane is called :

(a) an entire function

(b) an analytic function

(c) simple function

(d) None of the above

3. Every polynomial of degree n has :

(a) no zeros (b) n zeros

(c) n poles (d) None of these

4. For the function $f(z) = \sin \frac{1}{1 - z}$, $z = 1$ is :

(a) removable singularity

(b) isolated essential singularity

(c) non-isolated essential singularity

(d) none of the above

5. A function which has poles as only its singularities in the finite part of the plane is said to be :

(a) an analytic function

(b) a meromorphic function

(c) an entire function

(d) None of the above

6. A transformation of the type $w = \alpha z + \beta$ $(\alpha \neq 0)$ and α and β are complex constants is known as :

(a) Translation (b) Magnification

(c) Rotation (d) None of these

7. Fixed points of the bilinear transformation $w = \frac{z-1}{z+1}$ are :

(a) i, i (b) $i, -i$

(c) $1, i$ (d) $1, 1$

8. The transformation $w = z$ is :

(a) Conformal (b) Isogonal

(c) Both (a) and (b) (d) None of these

9. The space H(G) of analytic functions of G is a :

(a) Metric space

(b) Complete metrix space

(c) Not necessarily complete

(d) None of the above

10. If a set $F \subset C(G, \Omega)$ is normal then $\overline{F}$ is :

(a) Normal (b) Compact

(c) Bounded (d) None of these

SECTION B **5×4=20**

(Short Answer Type Questions)

Note : *Answer the following questions*

Unit-I

1. If $f(z)$ is continuous in a simple connected domain D and let for every closed contour C in the domain D, $\int_C f(z)dz = 0$, then show that $f(z)$ is analytic is D.

Or

Expand $f(z) = \frac{1}{(z+1)(z+3)}$ in a Laurent's series valid for the region $1 < |z| < 3$.

Unit-II

2. State and prove fundamental theorem of algebra.

Or

Prove that all the roots of equation $z^7 - 5z^3 + 12 = 0$ lie between the circles $|z| = 1$ and $|z| = 2$.

Unit-III

3. Define residue of an analytic function at any point $z = a$. Find the residue of $\frac{1}{(z^2 + a^2)^3}$ at $z = ia$.

Or

Apply calculus of residues to show that :

$$\int_0^{\pi} \frac{1+2\cos\theta}{5+4\cos\theta \text{ (denominator } 5+4\cos\theta)} d\theta = 0$$

Unit-IV

4. If the rectangular region D in the z-plane is bounded by $x = 0$, $y = 0$, $x = 2$, $y = 3$, then determine the region D′ of the w-plane into which D is mapped under the transformation $w = \sqrt{2}e^{i\pi/4}z$.

Or

Find the Mobius transformation which maps the points $z_1 = \infty$, $z_2 = i$ and $z_3 = 0$ into the points $w_1 = 0$, $w_2 = i$, $w_3 = \infty$.

Unit-V

5. Show that (C(G, Ω), ρ) is a metrix space, when C(g, Ω) is the set of all continuous function from G to Ω.

Or

If F ⊂ C(G, Ω) is equicontinuous at each point of G, then F is equicontinuous over each compact subset of G.

SECTION C **10×5=50**

(Long Answer Type Questions)

Note : *Answer the following questions.*

Unit-I

1. If $f(z)$ is analytic within and on a closed contour C, and let a be any point within C, then show that :

$$f(a) = \frac{1}{2\pi}\int_C \frac{f(z)}{z-a} dz$$

Using above formula evaluate $\int_C \frac{z^2 - z + 1}{z-1} dz$ where C is the circle $|z| = \frac{1}{2}$.

Or

State and prove Taylor's theorem for Analytic function.

Unit-II

2. State and prove Maximum Modulus principle.

Or

Prove that one of the root of the equation $z^4 + z^3 + 1 = 0$ lies in the first quadrant.

Unit-III

3. Apply the calculus of residue, to prove that :

$$\int_0^\infty \frac{x^2 dx}{x^6+1} = \frac{\pi}{6}$$

Or

Apply the method of calculus of residues show that :

$$\int_0^\infty \frac{\log(1+x^2)}{1+x^2} dx = \pi \log_2$$

Unit-IV

4. Define Bilinear transformation. Show that the transformation $w = \frac{5-4z}{4z-2}$ transforms the circle $|z| = 1$ into a circle of radius unity in w-plane and find the centre of the circle.

Or

Define conformal mapping. If $f(z)$ be an analytic function of z in a domain D of the z-plane and let $f'(z) \neq 0$ inside D. Then the mapping $w = f(z)$ is conformal at all points of D.

Unit-V

5. State and prove Montel's theorem.

Or

State and prove Weierstrass's theorem.

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