G-3/377/22

Roll No. ....

III Semester Examination, January 2022

M.Sc.

MATHEMATICS

Paper II

(Partial Differential Equation and Mechanics-I)

Time : 3 Hours] [Max. Marks : 80

Note : *All questions are compulsory. Question Paper comprises of 3 Sections. Section A is objective type/multiple choice questions with no internal choice. Section B is short answer type with internal choice. Section C is long answer type with internal choice.*

SECTION A 1×10=10

(Objective Type/Multiple Choice Questions)

Choose the correct answer :

1. The differential equation satisfying the harmonic function is called :

(a) Laplace's equation

(b) Transport equation

(c) Heat equation

(d) Wave equation

P.T.O.

2. In a dynamical system of N-particles with K constraint equation, the degree of freedom is :

(a) 3N + K (b) 3N – K

(c) 2N – K (d) 2N + K

3. A particle of mass m moves on a smooth plane then the Lagrange's equations are :

(a) $m\ddot{x} = F_x$ (b) $m\ddot{y} = F_y$

(c) Both (a) and (b) (d) None of these

4. For all $x, y \in U$, $x \neq y$, $G(y, x) = G(x, y)$ is called :

(a) Poisson's formula for half space

(b) Symmetry of Green's function

(c) Derivation of Green's function

(d) Estimate on derivatives

5. Euler's Lagrange equations are :

(a) $u(x, a) = ax + f'(a)$

(b) $H(P, X) = q(P, X) - PL(q(P - X) X)$

(c) $u_x(x, t) + H(D_u(x, t)) = 0$

(d) $-\frac{d}{ds}(D_q L(\dot{X}(s), X(s)) + D_x L(\dot{X}(s), X(s)) = 0$

6. Which is called Poisson's Identity ?

(a) $[u + v, w] = [u, w] + [v, w]$

(b) $[u, (v, w)] + [v, (w, u)] + [w, (u, v)] = 0$

(c) $[u, v, w] = [u, v]\, w + v\, [v, w]$

(d) $\frac{\partial}{\partial t}[u, v] = \left[\frac{\partial u}{\partial t}, v\right] + \left[u, \frac{\partial v}{\partial t}\right]$

7. Solution for $n = 1$ of wave equation by spherical means is called :

(a) Kirchhoff's formula

(b) Poisson's formula

(c) D'Alemberts formula

(d) Cauchy's formula

8. Attraction of a thin uniform spherical shell with mass m and radius a at point on its surface is :

(a) 0 (b) $\gamma \frac{M}{a^2}$

(c) $\gamma \frac{M}{4a^2}$ (d) $\gamma \frac{M}{2a^2}$

9. The law of Newtonian attraction are :

(a) $2\gamma k\rho$ (b) $\gamma m\omega$

(c) $F \propto \frac{m_1 m_2}{r^2}$ (d) $2\pi\gamma\rho k\,(1 - \cos\alpha)$

10. The Poisson's equation are :

(a) $\int_S N dS = -4\pi\gamma M$

(b) $\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$

(c) $\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = -4\pi\gamma\rho$

(d) None of the above

SECTION B **5×4=20**

(Short Answer Type Questions)

Note : *Answer the following questions.*

Unit-I

1. State and prove mean value formula's for Laplace's equation.

Or

Derive the formula for non-homogeneous problem.

Unit-II

2. Write the solution of wave equation by spherical means.

Or

Write the solution for $n = 1$ D'Alembert formula.

Unit-III

3. There exist a constant C such that $|u(x, t)| \leq \frac{c}{t^{1/2}}$ for all $x \in R$, $t > 0$.

Or

Derive the formula for straightening the boundary.

Unit-IV

4. Derive the attraction of a circular cone.

Or

Find the shortest line on the surface of the sphere.

Unit-V

5. Show that the attraction of a uniform cylinder of hight h, radius a and density ρ at a point on its axis at a distance c from the end outside it is $2\pi\gamma\rho\left[h - \sqrt{a^2 + (c+h)^2} + \sqrt{a^2 + c^2}\right]$.

Or

Prove that a solid uniform hemisphere of radius a exerts no resultant attraction at a point on its arc at a distance from the centre given by the equation $12C^4 - 8a^3C + 3a^4 = 0$.

SECTION C **10×5=50**

(Long Answer Type Questions)

Unit-I

1. State and prove Poisson's equation.

Or

Derive the fundamental solution of Laplace's equation.

Unit-II

2. Write the solution of non-homogeneous problem of heat equation.

Or

State and prove mean-value property of the heat equation.

Unit-III

3. State and prove derivation of Hamilton's ODE.

Or

Solve the Hamilton Jacobi equation.

Unit-IV

4. To find the attraction of uniform solid sphere at an external or internal point P.

Or

To find the attraction of a thin uniform rod AB on an external point P. Let k be the cross–section of ρ be the volume density of the rod.

Unit-V

5. State and prove Laplace's theorem.

Or

State and prove Gauss theorem.

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